

# E. S. Artium Principia:

O R, 1490. 4. 22.

The Knowledge of the first Principles  
of the MATHEMATICKS made Easy  
and Intelligible to the meanest Capacity.

Design'd for the more regular and speedy Introduction  
of young *Beginners* into those *Sciences*.

The Whole being contrived with the utmost demon-  
strative Plainness, and contained under the following  
*Divisions*; viz.

1. *Geometrical Definitions* of Lines, Angles, &c. from a Point to a Solid.
2. *Preparatory Problems*; or the Rudiments of Practical Geometry.
3. *Geometrical Theorems*; or Speculative Geometry, with a new and easy Method of Demonstration.
4. The Construction or Fabrick of *Sines, Tangents, &c.* by Lines and natural Numbers. Also,
5. A brief and clear Explication of the Tables of Logarithms of absolute Numbers, and the *Logarithmical Sines, Tangents, &c.* together with their Use and Operation in the *Mathematicks*.
6. *Trigonometry*; or the Doctrine of *Triangles*, both Plain and Spherical, performed three several ways, viz. *Arithmetically, Instrumentally, and Geometrically.*

Wherein are shewn the Mistakes of divers *Trigonometrical Writers*.

*To which is added,*

Something of the Nature of Proportion, and the chief Grounds  
and Reasons for varying thereof.

The Whole illustrated with Variety of suitable Schemes, and  
practical Observations.

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By HENRY BOAD.

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*Nisi fundamenta fideliter jeceris, quicquid superstruxeris corruet.* Quint,

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R. Hett, and T. Hatchett. M DCC XXXIII.



# Principia

The Knowledge of the Principles  
of the Mind and its Operations  
and the Principles of the  
Human Body and its Operations  
of the Human Body and its Operations

The Human Body and its Operations  
and the Principles of the  
Human Body and its Operations

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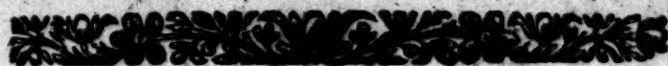
The Human Body and its Operations  
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and the Principles of the  
Human Body and its Operations





T O  
**JOHN GURDON,**  
Of ASSINGTON,  
AND

**JOSEPH ALSTON,**  
Of *Edward-Stone* in the County  
of *Suffolk*, Esq<sup>r</sup>.

GENTLEMEN,



YOU gave a Stranger so  
kind a Reception and  
Encouragement, at  
his first Appearance  
in those Parts, that he's even de-  
lighted with a Pretence to make  
you the Offer of a *small Present*,  
as a mean Testimony of the great  
Duty and Obligation which he  
bears toward you. And tho' the  
Tender here made you is so little  
A 2      worthy



## DEDICATION.

worthy of your Acceptance, that the Author blushes at the Thoughts of presenting it; yet he hopes that, small as it is, you will receive it with your accustom'd Goodness, as an Offering of *Esteem, Respect, and Gratitude.*

'Tis not for me, *Gentlemen*, to pretend to draw your Characters, which are too conspicuous to stand in need of it, and too illustrious for my Pen. Besides, I know you love to be concealed, and that you do not your Kindnesses out of Ostentation, but from a truly noble Principle of Piety towards God, and Love and Generosity to Mankind; and had much rather be publickly *useful*, than openly *admired*. All I shall desire is, that I may be allowed to confess  
the



## DEDICATION.

the Honour you did me, in permitting me to instruct some of the tender Branches of both your Houses, in that Part of Education which lies more immediately within the *Verge* of my *Profession*; and that you were pleased so far to approve of my Endeavours for that Purpose, as not only to recommend me to others *vivâ voce*, and by writing; but also one of you was so indulgent, as to take a Journey in my Behalf, on purpose to procure for me a comfortable Subsistence thereabouts; which was an Honour I was little aware of, and less deserving, as indeed were all the rest of your Kindnesses towards me: And tho' in the meanwhile an Offer of the like nature, that was then look'd upon as more



## DEDICATION.

beneficial, was made me from another Quarter, and which, with your Leaves, I accepted of; yet my Obligations to you remain uncancell'd, and will, I hope, be perpetually commemorated with all due *Gratitude and Thankfulness*.

I am very sensible, that the Faults and Imperfections of this Treatise will be but too apparent to Persons of your Judgments and Penetration; yet still I trust that Goodness and Beneficence, whereof I have already had so abundant Experience, will both excuse whatever is amiss in it, and also pardon the Presumption of this Address, from,

GENTLEMEN,

*Your most Obliged,*

*most Obedient, Humble Servant,*

HENRY BOAD.



# P R E F A C E.



*THE Design of writing the following Sheets, was not to assist those who are capable of profiting themselves by the more learned Treatises of this kind that are abroad in the World: But having, for some Tears past, been conversant in the Study and Exercise of Mathematicks, and finding by Experience the great Want of some very plain and familiar Book of the Rudiments of Mathematicks, which might serve to lead the Unlearned, and even the weaker Part of Mankind, into the Knowledge of these Things; I have, as well for my own*

own Ease in teaching, as to gratify several of my Friends, who seem pleas'd with my Method of Institution, drawn up these few Sheets, which I have cast into such a Form, as appeared to me most likely to place every thing in that Light, as would render it most easy and obvious to the meanest Capacity, and such as have not much Time to spare, and yet would willingly know something of Mathematicks, if they could be inform'd without the tedious and unavoidable Expence of Time and Money, which the looking over many Books, and searching here and there, must necessarily occasion.

My Desire of explaining every thing contained herein to the lowest Apprehensions, will, I am sensible, be censured by those, who cannot bear that any should be Partakers with them in the Knowledge they have acquired, as well as lay me under the Obloquy of the Captious and Ill-natur'd, who will be apt to condemn it, as a very mean Performance. And  
perhaps

perhaps I shall be told by them, that some things in it are not sufficiently exact, according to the Degree of Mathematical Certainty; namely, some of the Proofs in the Theoretical Part, as being too Mechanick to afford that full Conviction and Evidence to the Learner, which the Mathematicks, above all other Sciences, require; and consequently will not serve the Purpose for which they seem intended. But in answer to such Objection, tho' it may be sufficient to reply, That I propose nothing more by this Way, than to give the unskilful Learner some such general Ideas and Conceptions of these Matters, as may enable him to apprehend, with the greater Ease and Certainty, those Demonstrations which are more nice and accurate afterwards, if he has Leisure, and thinks fit to pursue these Studies; I may also add the Testimony of Experience, which daily convinces me, that Lads may be brought to a tolerable Acquaintance with the Elementary Parts of Mathematicks by this means, who are  
 unca-



untapable of understanding them by  
 any other Method whatsoever, at least  
 that ever I could find out: And 'tis  
 no matter how mean the Project is, if  
 it be useful, and serves to convey to  
 us the Knowledge of what we are  
 seeking after, tho' not in the strongest  
 and most perfect Light, as will better  
 be perceived by Learners when they  
 come further on. Provided always,  
 that we do not thereby imbibe any false  
 Notions of these Things, which we  
 shall be obliged afterwards to unlearn,  
 in order to our more regular Advance-  
 ment herein for the future. I ever  
 thought the plainest way of doing any  
 thing to be the best; and 'tis certainly  
 so in teaching any Art or Science; be-  
 cause it saves a Waste of Time, which  
 were much better employed in attain-  
 ing of other commendable Branches of  
 Knowledge, even in Boys of the  
 quickest Parts, but more especially in  
 such as are of slower Apprehensions,  
 who have need of all the Watchful-  
 ness, Care and Diligence of the Teach-

er, to fit them even for the common Concerns and Occurrences of Life.

It cannot, with any Shew of Reason or Judgment, be objected against this Work, That the same things are to be met with in other Books of Mathematicks now extant, and therefore this Performance is needless. For how can it be otherwise, unless a Person shou'd be supposed so very weak and absurd, as to pretend to coin new Elements more useful and better than those we have already; a thing impossible to be done, and therefore ridiculous to attempt. Besides, it should be considered, that tho' these things have been done by others before, they lie mostly out of the way of the common Sort of People, being generally in large Volumes, as Systems of Mathematicks, &c. which are composed chiefly for the Use of skilful Persons, and consequently are of little Service to such as these, as being above the reach of their Pockets. as well as their Capacities. Or else if they happen to be compriz'd in Volumes of a  
 lesser

lesser Size, they are then so short and superficial, that a Learner of common Abilities can reap but small Advantage from them. And 'tis not the Elements themselves, properly speaking, which fall under our Cognizance in this place, but the Manner of disposing and explaining them; which is not limited to any one particular Method whatever, but may be varied according to the Skill and Discretion of the Artist, who is left at his own Liberty herein; and accordingly they are render'd by some Authors much more familiar and easy to be understood than by others, who, it may be, rather affect to set forth their own Acquirements, than to inform the Judgment of their Readers. What I have here done, is with a quite different View to these last mention'd Writers; and with Design to try, if by any means I might render these things still more obvious to ordinary Apprehensions, and thereby make them to become more generally useful to the several Ranks and Degrees of Persons among us, to the Generality

nerality of whom they seem at present to be much obscured.

I am abundantly satisfied from my own Experience in teaching others, that nothing of this kind can be contrived too plain for the greater Part of Learners, who take in these things, at first, not without much seeming Difficulty, even when they are explain'd to them in the most easy and simple manner from the Mouth of a skilful Teacher, but much more so when they study them by Books only; and have therefore done my best Endeavours to familiarize these Matters so, as the weakest may be able to apprehend them without Difficulty; and those of quicker Parts need not be ashamed to learn by them, since they will often administer to them such useful Hints, as may be of service in their further Advancement in these Studies.

And, as I have all along had an eye to the Benefit of the meanest Capacities in composing this Work, more than my own Advantage, I freely sub-



mit it, such as it is, to the Candour and Judgment of the Ingenuous, without any farther Apology or Excuse: Only let me add a few Words more, for the Learner's Satisfaction, concerning the Nature and Design of it; after which I shall leave him to consult his Reason, and, if he pleases, his Experience, in order to judge of its Usefulness; and if, upon Trial, it be found to serve as a useful Introduction to his farther Enquiries into these sublime Studies, 'tis all the Author intended by writing it.

The Title of the Book manifestly points out the Persons, for whose Service, chiefly, it was intended, viz. those who understand nothing of Mathematicks, and would willingly begin to learn it the safest, best, and cheapest way: But may also be of service to such as have learnt the Mathematicks before, but are not so well grounded in the Foundation as they ought; and therefore contains such Rudiments, or first Principles, as are of general Use throughout most Parts thereof,

thereof, and which ought most industriously to be inculcated into the Minds of Beginners, at their first entrance upon these Studies. And to make these Things as easy and intelligible as possible, I have begun with a Point in Geometry, which is the Beginning of Magnitude, and may be conceived so small, as to have no Dimensions at all; and from thence to the several Definitions and Properties of Lines, Angles, &c. to a Solid, which is a Magnitude of three Dimensions, viz. Length, Breadth, and Thickness. After these, I have set down and explained such leading or preparatory Problems and Theorems in plain Geometry, and other things, in the Order mention'd in the Title Page; and therefore need not be here repeated, as are absolutely necessary to lay a good Foundation for the young Mathematician to build upon, proceeding gradually from the easier to the more difficult Matters; which must, I am sure, be the only safe way to instruct a Beginner in his Progress and Advance-

ment in any Art or Science whatsoever. Learners will be pleased, when they can with Ease and Judgment proceed from one thing to another, by having them proportion'd to the Meanness of their Understanding; and will by this means be led, as it were, insensibly into the farther Pursuit of these Studies, whose Curiosity might otherwise never have been prompted to any deeper Researches into these Matters, or even to take a Review of them with Pleasure, if they had found themselves too much puzzled and perplex'd with them at first; and Teachers will find their Advantage in it, when they come to account for the Improvement of their Pupils. For they who set out right at first, and are well instructed in the Fundamentals of an Art, will not be altogether unprepared for Examination, or to proceed in the Study of any Part thereof, but must needs make their Advances therein with much more Pleasure and Certainty. Every Step that such a Learner takes, opens a new and pleasant

*sant Prospect to him, into which he looks with an agreeable Surprize and Satisfaction; as being sure to shun all those Doubts and Difficulties, which another, who is not so happy, meets with almost at every turning, and so get to his Journey's End much sooner: Whereas he who starts wrong at the Beginning, has got his Ground to go over again, if ever he intends to reach the Goal, and win the Prize. Nor can we better account for the Miscarriage of so many, who begin to study Mathematicks, and are not able to make any considerable Progress therein, than that they labour in the wrong Place, and so their Pains turn to no Advantage. But were the contrary Method used; and would such as pretend to study Mathematicks from Books begin at the Foundation, there would not, I am confident, be so many conceited, ignorant Pretenders to it, as are now to be met with in almost every place; who fancy themselves, or at least would be thought by others, to understand those things, if they can*

( a 3 )

but



*but tell what Longitude and Latitude means, in the vulgar Sense of the Terms, and are able, it may be, to make an ordinary Dial to shew the Time of Day; to survey a Piece of Land, and perhaps to perform some other such like inconsiderable Feats, tho' they know nothing of the Ground-work, nor can they render any Reason for what they do, any more than this, That so their Book taught them; and therefore they are sure all is well, if they have but done as that directed.*

*And because I look upon a thorough Knowledge and Acquaintance with the Doctrine of Triangles to be of great Importance in the Study of Mathematicks, there being hardly any of those Arts whatever that can well be supported without it: To instance in Opticks, Perspective, Fortification, Gunnery, Navigation, Dialling, &c. most of the Problems belonging to each of them being beholden to Trigonometry for their Solutions. And what a mean and contemptible Figure would the noble Science of Geometry, Cosmography,*

inography, Geography, Astronomy, &c. make in the learned World, if it were not for Trigonometry? How should we be able to come at the Knowledge, either of the Heavens, or of the Earth, without its Help and Assistance? I have therefore made it a considerable Part of this Work, and taken more than ordinary Pains to explain and accommodate it to those of tender Years and Capacity, by accounting for its Principles (after due Preparation aforehand to qualify the Learner for it) in the most plain and rational manner; and rendering the Examples to every Case as easy and intelligible as possible, both Arithmetically, Instrumentally, and Geometrically. And that the Learner may the better perceive the Nature of his Work in the geometrical Solution of the several Cases of Sphericks, I have first of all laid down and explain'd such Affections or Properties of the Circles of a Sphere, as are applicable to Spherical Trigonometry; and have also given Directions how to project  
any

any Spherical Angle, according to the Number of Degrees proposed; and likewise the Manner of projecting any greater or lesser Circle belonging to the Sphere, in whatsoever Position it be required, and to find its Pole, and measure any Part thereof; as also how to measure any Spherical Angle. Which Things being well understood, the whole Doctrine of Sphericks, after this manner, will be very easily performed. I could wish Learners would be so kind to themselves, as, for their own Advantage, to practise this way of solving all the Cases of Trigonometry, which would give great Light into the Arithmetical Operations; and that they would be very careful to learn the whole of this Art of Trigonometry, and not let any Part of it slip, but endeavour to understand it perfectly, before they proceed to the Study of any one Branch of Mathematicks whatever. Then would every Step they take be attended with Success, and they themselves become so good Proficients, as to be able

able to give a demonstrable Account of what they perform; which they who fall directly upon the Study of an Art, without ever acquainting themselves with its Fundamentals, can never do; but are left to grope out their way in the dark, where they often miss it, and fall into many gross Errors and Mistakes. But to return from this small Digression, to what I was speaking of the Book. I have also attempted to make the Mysteries of Proportion, which I take to be an essential Part of Mathematicks, easily comprehended by an ordinary Capacity, from an Example, which, in my Opinion, will render these things very intelligible, that most young Beginners are so apt to be discouraged at. And because my Aim has been to bring this Work into as small a Compass as I could, to prevent the Excuses of such as are not willing to be at the trouble of turning over a bulky Volume, or to part with but little Money, as well as for the Encouragement of those who are not able; I have



have been obliged, throughout the whole, to use a sort of Abbreviations to express my Meaning; but they are so few, and so obvious, that I hope no one will have Reason to complain of Obscurity on this, or any other Account.

I am very well aware of the intolerable Trouble and Interruption sometimes given to Learners, by consulting a large Number of small Schemes placed upon half Sheets to fold out; where, besides the Disadvantage of casting the Eye to find out the particular Figure they want every time they have occasion to look upon it, it's a great chance if, where there are many of them together, some of them are not number'd wrong, either thro' the Author's, or the Engraver's Mistake; or perhaps the Printer has misplac'd some of the References: Either of these Failings may, and do frequently happen; which must needs create an infinite deal of Vexation and Trouble to a young Beginner, who cannot be suppos'd to help himself by  
judging

*judging of the right, but must be left to very great Uncertainties about the matter. I have therefore, to remedy this Inconvenience, ordered all the Schemes in the Margin along with the reading, (except three or four) that so the Eye may the more readily be fixed upon them, and the Danger of mistaking the Scheme avoided.*

*To conclude: If any small Mistake has happen'd in the whole Course of this Performance, which I am not aware of, I hope the Candor of the Reader will excuse it, and make every proper Allowance that may be due to a Work of this nature; or the frequent Interruptions, Discomposures and Discouragements of the Writer, during the greatest Part of the Time these Sheets were under his Hand.*

( 178 )

King the 2nd of March.



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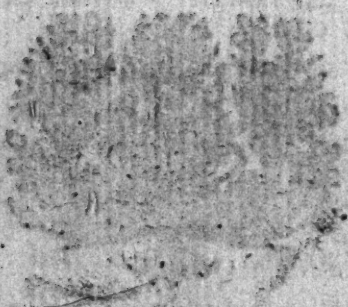


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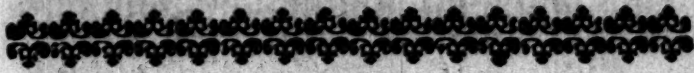
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## Advertisement.

**A**NY young Gentlemen may be instructed by the Author, at his House in *Colchester*, in *Writing*, *Arithmetick*, and *Merchants Accompts*, after the newest and most approved Method; and now used and practised by the best Masters. Where they may be also expeditiously taught the *Law-Hands*, with the usual Abbreviations belonging thereunto; and compleatly fitted for *Clerkships*, or other *Employments*. And likewise may be learnt the following Branches of the *Mathematicks*, viz. *Geometry*, *Astronomy*, *Geography*, *Trigonometry* Plain and Spherical; *Fortification*, *Gunnery*, *Navigation*, *Dialling*, *Land-Measuring*, &c. the Use of the *Globes*, *Sphere*, *Quadrant*, *Sector*, and other *Mathematical Instruments* now in Use.

## E R R A T A.

**I**N Page 13. the Scheme is not drawn true, the several Arches thereof not answering to the Divisions of the Circle within as they ought; but the matter is so plain, that the Learner may make the Experiment by any other Figure whatsoever. Page 75. in the Note at the bottom, read page 54. P. 81. in the Note at the bottom, for *Indexes* r. *Indices*. P. 90. l. 20. dele *as per Corol. 2. to Theorem 5.* P. 95. l. 32. after *I*, dele *by this first Axiom*, and add it after *easy*. P. 118. l. 6. for *Side*, r. *Sides*. P. 161. l. 2. dele *add*. P. 193. for *Case 2.* r. *Case XI.* P. 219. l. 19. for *show*, r. *shown*. P. 222. l. 9. r. *cf.  $\angle DCE$ .* l. 14. r. *cf.  $\angle DCE$ .* P. 235. l. 9. for  *$\angle$ 's C and D*, r.  *$\angle$ 's C and B.* P. 274. l. 17. r. *Fig. 6.*

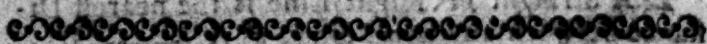




# *Artium Principia:*

O R,

The Knowledge of the first Principles of the MATHEMATICKS made easy and intelligible to the meanest Capacity.



## C H A P. I.

*Containing such General Maxims as are absolutely necessary to be first understood by a Beginner, in order to lay a good Foundation in the Mathematicks.*

## S E C T. I.



**B**ECAUSE the very Title of this Book supposes the Person who uses it to be ignorant of the first Principles of this Art, I have begun with the Explanation of such general Terms, Definitions, &c. as no one who pretends to study Mathematicks, ought to be unacquainted withal; and have ended



endeavoured to do it in the most plain and easy Method I was capable; and so to proceed with the whole. And first, to begin with the *Definition*

*Of Lines and Angles.*

1. A Point is that which is considered as having no manner of Dimensions, *i. e.* of Length, Breadth, &c. being perfectly indivisible as the Point A.

2. From the Motion of a Point is generated a Line, which is called a Quantity of one Dimension, because it may have any Length, but no Breadth nor Thickness, as the Line A \_\_\_\_\_ B made by the Motion of the Point A going forward to B.

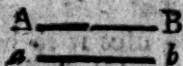
3. Of Lines there are two Sorts, *viz.* Straight and Crooked, or Curved Lines.

4. A Straight or Right Line is the nearest Distance between any two Points, as A \_\_\_\_\_ B.

5. A Crooked or Curved Line is that which lies bending between those two Points which limit its Length, as the Line CD or FE, &c.



6. Parallel Lines are those which lie equally distant from each other in all their Parts, and being continued infinitely would never meet or cross one another in any Point, as the Lines AB and *ab*, or CD and *cd*.



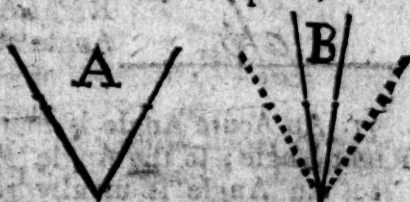
7. Any

7. Any two Straight Lines falling one upon the other, do constitute or form what Geometricians call an Angle, as the Lines A B and A C; the Meeting whereof, viz. at A, is the angular Point, and



is said to be less or greater, as the Lines by which it is made are nearer or wider asunder, whether the Lines which include the Angle be long or short: Thus the Lines A b and A c include the same Angle as A B and A C, notwithstanding they are shorter. And this should be carefully heeded, because 'tis a Mistake which young Beginners are apt to fall into. But to make it more plain,

Let them imagine the Angle A to be laid upon B, as the pricked Lines about B do represent; then 'tis evident, the Angle B

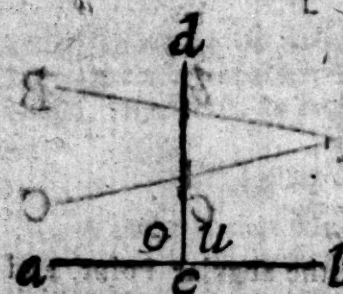


will easily be contained within A, and therefore is less than A, by how much its Legs come nearer, or are more inclined towards each other, than those of A; and consequently A will be a greater Angle than B, because its Legs are farther distant from each other, than the Legs of B.

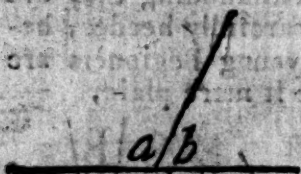
8. Angles are of three Kinds,

Viz.  $\left\{ \begin{array}{l} \text{A Right} \\ \text{An Obtuse} \\ \text{An Acute} \end{array} \right\} \text{Angle.}$

Note, The two last are in the general termed Oblique Angles.



9. A Right Angle is at the meeting of two Right Lines, when the one falls upon the other so directly, as that it neither inclines nor declines to one Side more than the other; which Lines so meeting, are said to be perpendicular to each other. Thus the Lines  $ac$ , and  $cb$ , are perpendicular to  $cd$ , as well as  $cd$  is to either or both of them: And the Angles  $o$  and  $u$ , are both Right Angles.



10. An Obtuse Angle is that which is greater than a Right Angle, and such is the Angle at  $(a)$ .

11. An Acute Angle is that which is less than a Right Angle; so the Angle  $(b)$  is said to be Acute.

12. An Angle is usually represented by three



Letters, whereof the middlemost always signifies the angular Point; as the Angle  $a, b, d$ , is the Angle which by the Lines  $b, a$ , and  $b, d$ , is formed at the Point  $b$ .

But I chuse rather to express it thus,  $a(b)d$ .

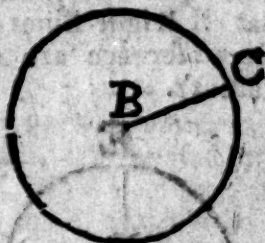
13. An Arch of a Circle is the Measure of an Angle, being described upon the angular Point; so the Measure of the Angle  $c(b)e$ , is the Arch  $c, e$ .

What an Arch of a Circle, and the rest of its Parts are, see in the following Section.

## S E C T. II.

### Of a Circle, &c.

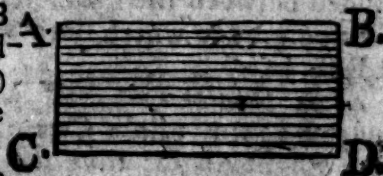
*Def.* **B**Ecause a Circle is a plain regular Figure, which by one continued Line, called the Circumference or Periphery, does circumscribe a certain Space, and set Bounds or Limits thereunto, as the Figure B: It will, I conceive, be convenient, before we go any farther, to say somewhat briefly of Superficies in general.



Mr. *Ward*, in his Introduction to the *Mathematic*. pag. 285. defines it thus:

1. A Superficies or Surface is the upper or very Outside of any visible Thing. But by Superficies in Geometry, is meant only so much of the Outside of any thing as is inclosed within a *Line* or *Lines*, according to the Form or Figure of the thing designed, and is produced or formed by the Motion of a *Line*, as a *Line* is described by the Motion of a *Point*, thus:

Suppose the Line AB were equally moved (upon the same Plane) to CD; then will the Points at A and B describe the two Lines AC and BD; and by so doing, they will form (and enclose) the Superficies or Figure ABCD, being a Quantity of two Dimensions, viz. Length and Breadth.



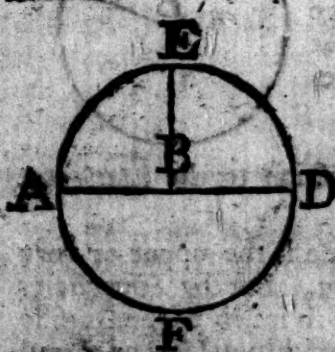


*Breadth*, but not *Thickness*. Consequently the Bounds or Limits of a *Superficies* are Lines.

*Note*, The *Superficies* of any Figure is usually call'd its *Area*.

*Note* also, That the Line *BD*, in the Figure above, is call'd the *Radius* or *Semidiameter*; the End whereof, *viz.* at the Point *B*, is the Centre of that Circle.

From the foregoing Definition of a Circle, it will, with a very little Consideration, be evident, that all *Right-Lines* drawn from the Centre to the Circumference are equal; because they are all *Radius's*.



2. The *Diameter* of a Circle is a *Right Line* drawn through the Centre to either Side of the Circumference, dividing the Circle into two equal Parts, and is equal to twice *Radius*, as the Line *ABD*.

Hence 'tis plain, that those Circles which have one and the same *Radius* are equal; for equal *Radius's* must describe equal Circles.

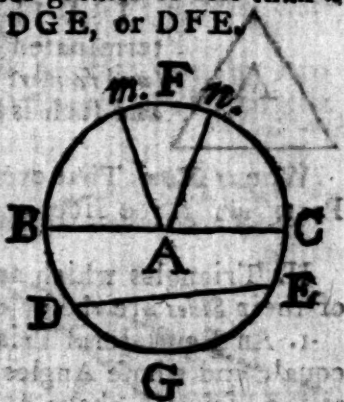
3. A *Semicircle* (or half Circle) is a Figure included between the *Diameter* and that Part of the Circumference which lieth on one side thereof, as *AED*.

4. A *Quadrant* is half a *Semicircle*, or one quarter of the whole Circle, and is the Measure of a *Right Angle*; thus the Lines *BE* and *BD* do contain  $\frac{1}{4}$  Part of the Circle *AEDF*, which is also the Measure of the *Right Angle EBD*.

5. A *Segment*, *Section*, or *Portion* of a Circle, is a Figure contained under one *Right Line*, and a Part of

of the Circumference either greater or less than a Semicircle, as the Figure DGE, or DFE.

6. A *Sector* is a Figure included between two Right Lines, or Radius's of the Circle, and an Arch of the Circumference, cut by the said Lines: Or it is an Angle at the Centre contain'd under two Semidiameters and a Part of the Periphery, as the Figure Amn.



Here *Note*, The Periphery (or Circumference) of every Circle, great or small, is supposed to be divided into 360 equal Parts, call'd Degrees; and every of those Degrees are again supposed to be divided into 60 others, call'd Minutes, &c. So that

A Semicircle contains 180 Degrees, and  
A Quadrant 90 Degrees.



### SECT. III.

#### Of Triangles.

AS there are three several Sorts of Angles, which has been already shown, (Per 8. Sect. 1.) so there are also as many kinds of Triangles which borrow Names, either from their Sides or Angles; as we shall make appear when we have first given the Definition of a Triangle.



I. A Plain Triangle is a Figure terminated by three Right Lines, (and no more) making as many Angles; and such is the Figure A.

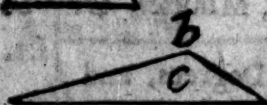
Where *Note*, That every Triangle consists of six Parts, viz. three Sides and three Angles.

II. Triangles which are distinguished in respect of their *Sides* are these, viz.

1. An Equilateral Triangle, whose Sides are all equal, and whose Angles contain each 60 Degrees, as the Figure A above.

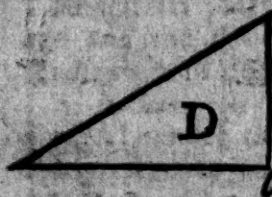


2. An Isosceles, or Equicrural Triangle, whose two Sides only are equal; as B.



3. A Scalenous Triangle, which hath all its three Sides unequal, as C.

III. Triangles which are distinguished with respect to their *Angles* are,



1. A Right-angled Triangle, which hath one Right Angle, as D, whose Angle at (a) is a Right Angle, i. e. it contains just 90 Degrees.

2. An Obtuse-angled Triangle, call'd also an Amblygonium Triangle, which hath one of its Angles obtuse (or blunt) i. e. greater than a Quadrant or 90 Degrees, as the Angle b, in the Figure C above.

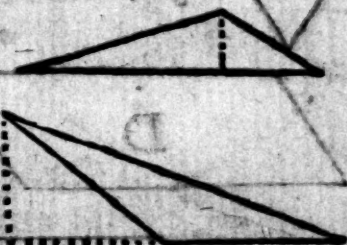
3. An Acute-angled Triangle, call'd likewise an Oxygonium Triangle, whose three Angles are all Acute.

Acute (or Sharp) i. e. less than 90 Degrees, and such is the first Triangle B.

*Note* here; (as before of Angles) that the two last of these are in general Terms call'd Oblique Triangles.

4. The Perpendicular Height or Altitude of any Plain Triangle, is the Length of a Right Line let fall from any of its Angles to a Point directly opposite thereto

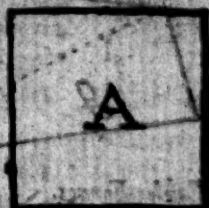
in one of its Sides, produced if need be. See the prick'd Lines in the answer'd Triangles.



## S E C T. IV.

Of Quadrilateral or Four-sided Figures, &c.

1. A Square is a plain regular Figure, whose four Sides are all equal, and its Angles Right Angles, as A.



2. A Rectangle, or Right-angled Parallelogram (call'd also an Oblong), and sometimes (vulgarly) a Long Square, is a Figure whose Sides are unequal, but

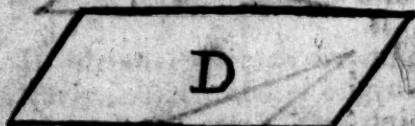


its Angles right as B.

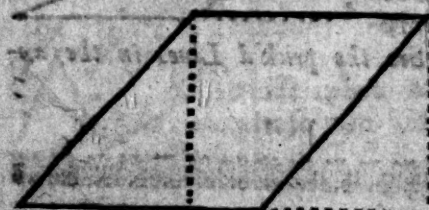




3. A Rhombus, or Diamond-like Figure, is that which has equal Sides, but unequal Angles, as C.



4. A Rhomboides hath both its Sides and Angles unequal, as D.



5. The Perpendicular Height or Altitude of any Oblique - angled Parallelogram, viz. either of the Rhombus or Rhomboides, is the same as above, (in Def. 4. of Sect. 3.) of a Plain Triangle, &c. as is evident by the Pricked Lines in the annexed Figure.



6. All Four-sided Figures, which fall not under the Consideration of one or other of those before named, are call'd Trapezia, or

*Table Forms.*

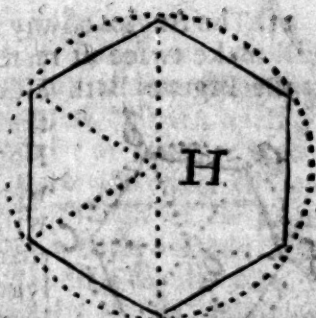
That is, when neither their opposite Sides nor Angles are equal; as the Figure F.

7. A Right Line being drawn from any Angle in a Trapezium to its opposite Angle, is call'd a Diagonal, and will divide it into two Right-lin'd Triangles, viz. F and g.

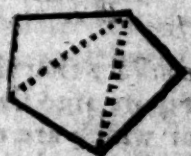
8. All Right-lin'd Figures of more than four Sides, whether they be regular or irregular, are in general termed Polygons, i. e. having many Corners: for so the Word signifies.

9. A

9. A regular Polygon is that whose Sides and Angles are all equal one to another, as the Figure H.



10. An Irregular Polygon hath both its Sides and Angles unequal like the annexed Figure.



Of this Kind of Polygons there are infinite Varieties. But of the other, viz. the Regular Polygon, there are usually reckon'd six, which, for Distinction sake, are named according to the Number of Sides (or Angles.) Thus, if it hath

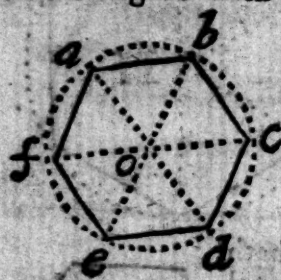
|                                                                             |                    |                                                                                                                                                                               |
|-----------------------------------------------------------------------------|--------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\left\{ \begin{array}{c} 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} \right\}$ | Sides, 'tis call'd | $\left\{ \begin{array}{c} \text{A Pentagon.} \\ \text{A Hexagon.} \\ \text{A Heptagon.} \\ \text{An Octagon.} \\ \text{A Nonagon.} \\ \text{A Decagon.} \end{array} \right\}$ |
|                                                                             |                    |                                                                                                                                                                               |
|                                                                             |                    |                                                                                                                                                                               |
|                                                                             |                    |                                                                                                                                                                               |
|                                                                             |                    |                                                                                                                                                                               |
|                                                                             |                    |                                                                                                                                                                               |

*Note,* All Regular Polygons (how many soever the Sides and Angles are) may be inscrib'd within a Circle, i. e. their angular Points will all just touch the Circumference or Periphery; and all Polygons whatever may be divided into Triangles, by drawing Lines from a Point within to every Angle, or from any one Angle to all the rest.

But in this last Case, the Number of Sides will always exceed the Number of Triangles drawn within the Area, which in the other will be just equal thereto, as the prick'd Lines in the two Figures above will clearly demonstrate.

11. The

11. The Angles of any Polygon taken all together, will make twice as many right ones, wanting four, as the Figure hath Sides. Thus, for Instance, if



the Polygon has six Sides, the Double of that is 12; from whence subtracting 4, there will remain 8. Now, I say, that all the Angles of that Polygon, viz. *a, b, c, d, e, f*, taken together, are equal to eight Right Angles; which will appear very plain, if we

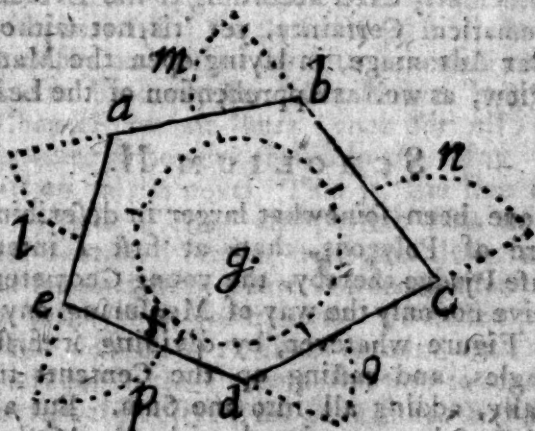
consider that the Lines *a, b, c, d*, &c. do divide the Figure into six Triangles; the three Angles of each of which are equal to two Right Angles (as shall be made obvious when we come farther on, viz. by *Theorem 3. of Chap. III.*) and consequently all their Angles taken together make twelve right ones. But now each of these six Triangles hath one Angle at the Centre (*o*); and from thence they compleat the Space all round the said Point; all which Angles taken together, are equal to four Right Angles: which is the same as to say, they are all equal to four Quarters of a Circle. Now each Quadrant, or fourth Part of a Circle, contains just  $90^\circ$ , or a Right Angle (as you have before learnt.) Consequently they must be equal to four right ones; for so many Right Angles do also compleat the Circle: Wherefore those ~~four~~ being taken from 12 (the Sum of the Right Angles of all the six Triangles) there remains 8, the Sum of the Right Angles of the Hexagon; which make in all 8 times  $90^\circ = 720$  Degrees; and therefore every Angle must be equal to  $\frac{2}{3}$  Part of that, viz. 120 Degrees.

So that 'tis plain the Figure hath twice so many Right Angles as Sides, wanting four; which was what we undertook to prove.

SCHOLIUM.

## SCHOLIUM I.

From a serious Consideration of the foregoing Premises, we may draw this undeniable Conclusion, viz. That all the external (or outward) Angles of any Right-lined Figure are equal to just four Right ones. And this, though it be plain enough to those who have but a little Skill in Mathematicks, I shall notwithstanding for the sake of the unlearned, make appear by a new and easy Method which serves to prove the Truth of many useful Propositions in *Euclid* and other famous Geometricians; as the Reader will perceive hereafter,



Let us then for Instance, suppose the Sides of the irregular Polygon  $a, b, c, d, e$ , to be lengthened out by the prick'd Lines, equal to the Radius of the Circle ( $g$ ) or of any other; and that with the same Radius there be described the several Arches  $l, m, n, o, p$ , from the angular Points of the Polygon, viz.  $a, b, c, d, e$ .



Now because every whole Circle is the Measure of four Right-Angles, (or  $360^\circ$ ) which I would have the Reader to remember: I say therefore that those prick'd Arches being taken severally, between the Compasses beginning at (a), and applied from f, to the several Marks on the Circle (g), will exactly Measure the whole Circumference as any one may by trial find. Consequently all the external Angles are equal to four Right ones, which was to be proved.

*Nota*, I advise my Reader to rest satisfied with this kind of Proof, till he can form to himself the Knowledge of a better; for though it be only Mechanical, and may be thought by the Curious not sufficiently exact according to the Evidence of Mathematical Certainty, yet 'tis not without its singular Advantage, in laying open the Matter to the View, as well as Apprehension of the Learner.

## SCHOLIUM II.

I have been somewhat larger in describing the Nature of Polygons, than at first I intended; because I judge thereby, the young Geometer will perceive not only the way of Measuring any irregular Figure whatever, by dividing it first into Triangles, and casting up the Contents thereof severally, adding all into one Sum. But also he may hence be acquainted with the Methods of composing the Tables of Sines, Tangents, &c. which we intend to say something of hereafter. Likewise, by inscribing Polygons in a Circle, may be understood the Way and Manner of Squaring the Circle, as exact as is needful; for we may imagine the number of Sides of any regular Polygon (described within a Circle) so small, as to be insensibly the same with the Circumference itself.

This

This is so plain that it needs nothing to be said about it, more than to put the Reader in mind to observe it.

12. A Solid, is a Quantity, which beside the foremention'd Properties of a Superficies, such as Length and Breadth, hath another also which is call'd Thickness or Depth, and may be fitly represented by the Figure *m*, whose Length is *no*, Breadth *ny*, and Thickness *1 n*.



I shall put an end to this Section with this Definition of a Solid, without mentioning any of the rest; because I think it not proper nor convenient in a Work of this nature, to *amuse* the young Beginner with any thing, that will rather confound than improve him; for these things will be much more easily understood by him hereafter, than is possible to be done now. But before I conclude the Chapter, let me here (once for all) advise the Learner to be very perfect in one thing, before he pass on to the next. That is, Let him endeavour to gain a clear and perfect Idea of every Matter contain'd herein, as he goes along, and not to rest satisfied with a bare Remembrance thereof or an Ability to perform them. To which purpose he would do well at all times to mind the Directions included in the Parentheses, and to consult the Places thereby referr'd to, for a fuller Explication of the Matter.





## C A A P. II.

*Preparatory Problems; or the Rudiments of plain and practical Geometry.*

**B**EFORE I begin the Work of this Chapter, it may not be amiss to explain some of those general Terms commonly made use of by Geometricians, &c. which are as follow.

1. A *Problem*, is that which relates to Practice, requiring something to be done; as to bisect a Line given; to draw a Circle through any three Points, &c. and does more immediately belong to practical than speculative Geometry.

2. A *Theorem*, is a Mathematical Declaration of certain Properties, Proportions or Equalities, duly inferred from some Suppositions about Quantity. That is, it does not require any thing to be done; but only declares, that if such an Operation be perform'd, from thence there will arise such and such Properties, &c. as if two Lines intersect or cut each other, the two opposite Angles will be equal, &c.

*Note*, *Euclid* and some others, include both these under the general Term of Proposition.

3. A *Reciprocal Theorem*, is that whose converse or contrary is true.

4. A *Covollary* or *Confectary*, is a Consequence or Deduction from a foregoing Argument or Proposition,

position, or from something that has been already proved or demonstrated.

5. A *Lemma* is a Proposition which serves previously to prepare the way for the Demonstration or Proof of some Theorem, or the Construction of some Problem.

6. A *Scholium*, is a brief Remark or Observation on any Proposition, or precedent Discourse.

7. As to *Axioms*, or *Postulates*, or *Petitions*, they signify only such easy and self-evident Principles, as need no Explication or Illustration to render them intelligible to those who have in the least consider'd them, and such are those that follow.

*Postulates or Petitions: Being the Ground-Work of all that is contain'd in this Chapter:*

1. That a Right-Line may be drawn from any one given Point to another.

2. That a Right-Line may be continued to any Length whatever.

3. That upon a given Point, or Centre, a Circle may be described at any distance required.

I do here advise the young Learner to acquaint himself with the Signification of the following Signs or Characters, not only because they will very much help to shorten the ensuing Work, as being a much clearer, better, and more significant Way of denoting what is to be done in most Conclusions, than can otherwise be express'd in Words at length. But also, for that being used by many of the best Authors in the Mathematicks, it will be a good Introduction and Help to the understanding of their Work.



*An Explication of the Characters.*

$+$  Plus } The Sign of Addition, as  $a + b$  is  $a$  or more. } added to  $b$ .

$-$  Minus } The Sign of Subtraction, as  $a - b$ , is  $a$  or less. } less by  $b$ , or  $b$  subtracted from  $a$ .

$\times$  Into } The Sign of Multiplication, so  $a \times b$ , is or with. }  $a$  multiplied by  $b$ .

$\div$  By. } The Sign of Division so  $a \div b$  is  $a$  divided by  $b$ , also  $a)b$  (or thus  $\frac{b}{a}$  all which signify the same thing.

$=$  Equal. } The Sign of Equality or Equation, so  $a = b$  is  $a$  equal to  $b$ .

$:$  So is. } The Sign of Proportion, as  $a : b :: c : d$ , signifies as  $a$  is to  $b$ , so is  $c$  to  $d$ . This Sign is always put between the two middle Numbers (or Quantities) in any Proportion.

$\angle$ , denotes an Angle, and  $\angle$ 's signifies Angles,  $\perp$  a Right-Angle;  $\triangle$ , signifies a Triangle;  $\square$  a Square; and  $\parallel$  a Parallelogram or long Square.

These Signs and their Significations being perfectly learnt, the Learner may proceed with the same Ease and Pleasure, as if all had been expressed in Words at large.

PROBLEM I.

A —————  
B —————

Two Right-Lines A and B being given, to find their Sum and Difference.  
(Euch. 1. 3.)

. Make

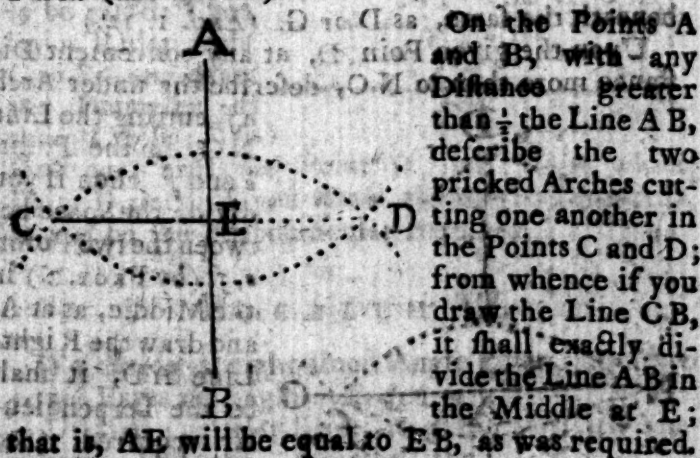
Make the shortest Line B Radius, and there-  
with describe the Arch DB, from the Center (o)



of which set off the other Line A, and join both in a Right Line. Then will  $AB = A + B$ ; and  $AD = A - B$ ;

## PROBLEM II.

To divide a Right-Line, as AB, into two equal Parts. (*Eucl. I. 10.*)



On the Points A and B, with any Distances greater than  $\frac{1}{2}$  the Line AB, describe the two pricked Arches cutting one another in the Points C and D; from whence if you draw the Line CB, it shall exactly divide the Line AB in the Middle at E; that is, AE will be equal to EB, as was required.

## PROBLEM III.

To a Right-Line given, as AB, to draw a parallel Line CD, that shall pass through any assigned Point, as at e; that is to say, at any Distance required. (*Eucl. I. 31.*)

Take

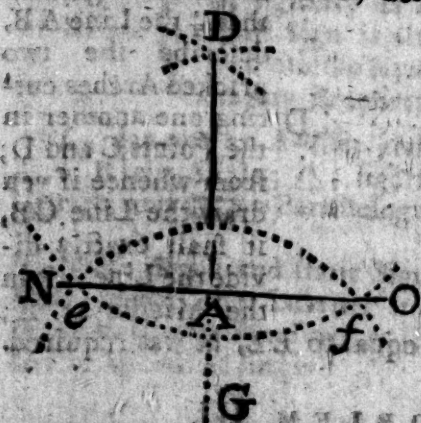
Take any convenient Point in the given Line A B, as suppose at F, (for the further from *e* the better) make *F* Radius, and with it describe the Semicircle G H M; then make the Arch G H equal to M *e*; and through the Points H and *e*, draw the Line C D; it shall be the parallel Line required. (See Def. 6. §. 1. Chap. 1.)



# PROBLEM IV.

To raise or let fall a Perpendicular, as A D, to a Right-Line N O, from a Point given above or beneath the same, as D or G. (*Eucl.* 1. 12.)

Upon the given Point D, at any convenient Distance more than to N O, describe the under Arch *e f*, cutting the Line N O, in the Points *e* and *f*. Then if you divide the Space between the two Points *e, f*, (by PROB. 2.) in the Middle, as at A, and draw the Right-Line A D, it shall be the Perpendicular required. (See Def. 9. §. 1. of Chap. I.)



The like Construction is to be used, if the Point were beneath the given Line N O, as the Figure plainly shews.

But if it had been required to erect a Perpendicular from A upon the Line N O; then upon the Point *e* and *f*, or any other equal Distances from A,

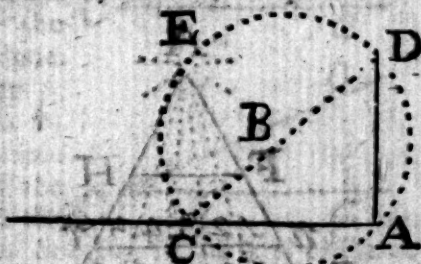
A, on both Sides, describe two Arch-Lines crossing each other in D; from which Point D, if you draw a Line to A, it will be, &c. as was required.

PROBLEM V.

To erect or raise a Perpendicular upon the End of any Right-Line given, as at A. (*Eucl. 1. 11.*)

Open the Compasses to any convenient Distance, and setting one Foot in the Point B, with the other draw the Circle A C E D, but so as that it may always pass

through the Point from whence the Perpendicular is to be rais'd: Then lay a Ruler from C to B, and draw the Diameter of the Circle;



so shall you have the Point D given; from which Point D to A, if you draw the Right-Line AD, it will be the Perpendicular required.

PROBLEM VI.

To divide a right-lined Angle given, into two equal Parts or Angles. (*Eucl. 1. 9.*)

Upon the angular Point, as at O, with any convenient Distance, describe the Arch F E; and from F and E draw the Arch-Lines Ff and Ee, crossing each other as at c: Then through the Intersections of those Arches, draw the Line cO, and it will bisect the Arch F E, and consequently the  $\angle$ , as was required.



PROBLEM



## PROBLEM VII.

To divide a Right-Line given, as OP, into any Number of equal or unequal Parts; or like to any other Right-Line propounded. (*Eucl. 6. 10.*)

The best Method for a young Beginner to perform this, I take to be as follows:

Upon any Right-Line drawn at Pleasure, as the pricked Line LM, with any small Opening of the Compasses, measure the same any Number of

times (suppose 5.)

Then take the Distance from O to

5, and make there- with an Equilate- ral  $\Delta$ , as ALM,

(by P. 0. 8. 9.) A- gain, take in your

Compasses the gi- ven Line OP, and

set from A, to  $\pi$  and  $\gamma$ . Then if

you lay a Ruler from A to the se- veral Points 1, 2,

3, and 4, in the Lines LM, it will cut the given

Line OP in the Points 1, 2, 3, 4, dividing it into

five equal Parts, as was required.

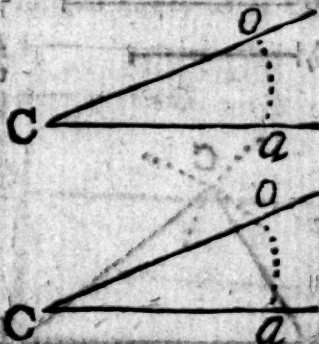
And thus may this Equilateral  $\Delta$ , be fitted for dividing any other Line greater or smaller, as FH and FS into five equal Parts; as the pricked Lines in the Figure will demonstrate: The same must be done, if the Parts had been unequal. But this is so plain, that I shall not spend further time about it.

PROBLEM

PROBLEM VIII.

To protract or lay down an  $\angle$  of any Quantity of Degrees, or to make an  $\angle$  equal to an  $\angle$  given. (*Eucl. 1. 23.*)

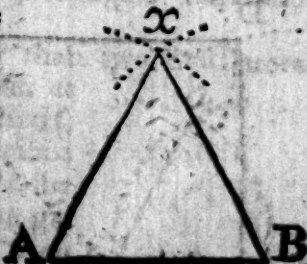
Let the  $\angle$  required be 40 Deg. First, with any Radius on the Point C, describe the Arch  $a, o$ , the Measure wherof let be 40 Deg. then through the Point  $o$ , draw the Line  $Co$ , and it's done. Thus may any other  $\angle$  be made like unto it, by using always the same Radius; for then if the Arches be equal, the  $\angle$ 's will be the same. See *Def. 13. of §. 1. of Ch. 1.*



PROBLEM IX.

To describe an Equilateral  $\Delta$  upon a given Right-Line A B. (*Eucl. 1. 1.*)

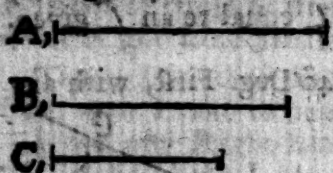
Take the given Line A B, in your Compasses, and placing one Foot in A and B, describe the two pricked Arches crossing each other in the Point  $x$ ; then join A  $x$  and B  $x$  with Right-Lines, and they will give you the  $\Delta$  required. See *Def. 2. Art. 1. §. 3. of Chap. 1.*



PROBLEM

PROBLEM X.

Three Right-Lines being given, to make there-  
of a  $\Delta$  (provided any two of them taken together  
be longer than the third.) (*Eucl. 1. 22.*)

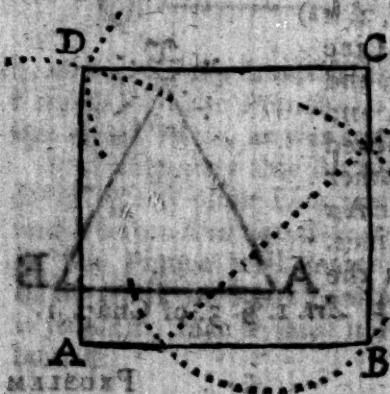


Take either of the shorter  
Lines, as C, in your  
Compasses; and on one  
End of the longest Line  
A, draw a small Arch;  
then take the other Line  
B; and placing one  
Foot in the other End  
thereof, describe ano-  
ther Arch to cross the  
former, as at C. Draw  
AC and BC; so is the  
 $\Delta$  completed, as was  
required. See *Def. 2.*  
*Art. 3. 1. 3. of Chap. 1.*

PROBLEM XI.

A Right-Line, as AB, given, to make there-  
with a true geometrical Square. (*Eucl. 1. 46.*)

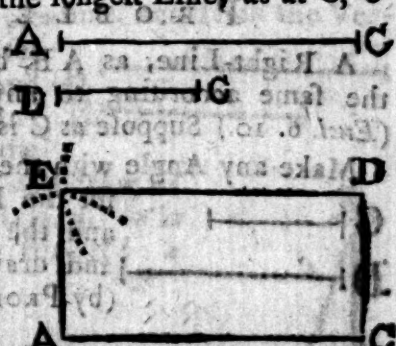
Upon one End thereof, as at B, erect the Per-  
pendicular BC, of  
the same Length  
with AB; and with  
that Distance, fixing  
one Foot of the  
Compasses in C and  
A, draw two small  
Arches crossing each  
other in the Point  
D; join AD, and  
CD, with Right-  
Lines, and they will  
constitute the Square  
required.



PROBLEM

## P R O B L E M XII.

Two unequal Right-Lines being given, to make thereof a right-angled Parallelogram, or Long-Square. This Problem I do not find distinctly laid down in *Euclid*; but it's well enough supplied by the last.

Upon one End of the longest Line, as at C, erect a Perpendicular  of the same Length with the shortest Line D, C: then make AC as a Radius, and describe therewith, from the Points D and A, the two prick'd Arches crossing each other in E: join A E and D E with Right-Lines, so shall you have the Parallelogram or Oblong required.

## P R O B L E M XIII.

Any Right-Line being given, to form or make thereof a *Rhombus*, or oblique-angled Parallelogram. (*Eucl.* 1. 17)

This Figure is no other than two Equilateral Triangles join'd Base to Base, as the prick'd Line *cd* plainly shews, and might be made or described after the same manner: (*viz.* by PROB. 9.) Or thus; Make the given Line Radius, and on each End thereof describe an Arch; which being continued till it meet with the other Arch in the Points *a* and *b*, shall give you the Length of the *Rhombus* sought: and if you



D

divide



divide the two Arches in the middle, which is done with the same Radius, it will help you to the Breadth thereof, viz.  $c d$ . And thus have we found four Points, from which drawing Right-Lines, they will complet the Figure required.

# P R O B L E M XIV.

A Right-Line, as  $A B$ , being given, to divide the same according to any Proportion assigned. (*Eucl. 6. 10.*) Suppose as  $C$  is to  $D$ .

Make any Angle with the given Line  $A B$ : then set the Line  $C$ , from  $A$  to  $n$ , and the Line  $D$  from  $n$  to  $b$ ; and draw  $n L$  parallel to  $b B$ , (by *PROB. 3.*) so shall the Point  $L$  divide  $A B$  in the Pro-

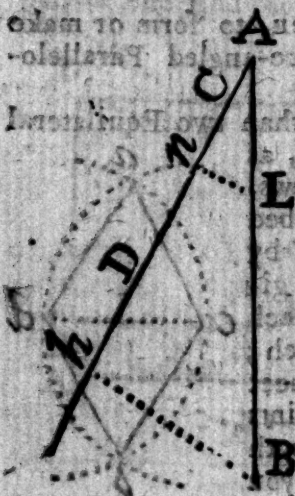
portion required. That is to say,

As  $C : D :: A L : L B$ .

For  $C + C = D$ ; so also,  $A L + A L = L B$ .

And much after the same Way may you cut off from the given Line  $A B$ , or any other, what Number of Parts you please: As, for instance, suppose  $\frac{2}{5}$ .

Make any Angle with the given Line  $A B$ , as before, and let  $A b$  be made equal to five Parts taken from any Scale: Then set two such Parts from  $A$  to  $L$ , join  $b B$ , and draw  $n L$  parallel to it: So is  $A L = \frac{2}{5}$  of  $A B$ , as was required.



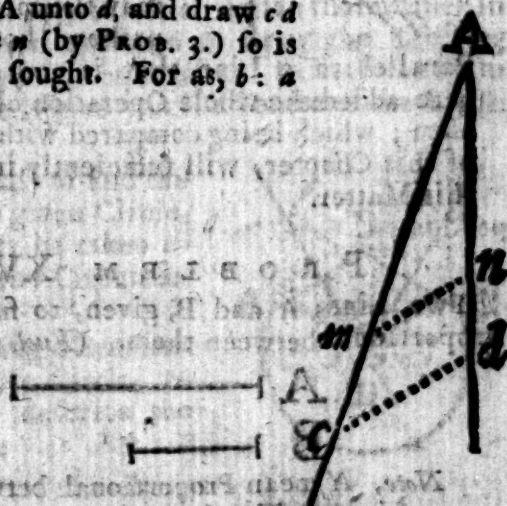
*Euclid* makes this a distinct Proposition from the last, entitling it, in the order of Places, his 9th of Book VI.

PROBLEM XV.

Any two Right-Lines, as  $a$  and  $b$ , being given, to find a third proportional to them. (*Eucl.* 6. 11.)

Make an Angle, at pleasure, and from the Vertex or Top thereof  $A$ , set the two given Lines  $a$  and  $b$  from  $A$  to  $m$  and  $n$ ; and draw  $mn$ ; place also the

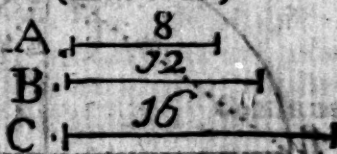
Line  $a$  from  $A$  unto  $d$ , and draw  $cd$  parallel to  $mn$  (by *PROP.* 3.) so is  $Ac$  the Line sought. For as,  $b : a :: a : Ac$ .



PROBLEM XVI.

Any three Right-Lines, as  $A, B, C$ , given, to find a fourth proportional to them: Or to work the Rule of Three by Lines. (*Eucl.* 6. 12.)

Make any Angle as before, and from the Point  $D$ , set the two first Lines  $A$  and  $B$  to  $o$  and  $n$ ; then set the 3d Line



$D \quad o \quad n \quad C \text{ from } D$

C from D to  $e$ ; draw the Line  $p, n$ , and through the Point  $e$ , draw  $er$  parallel thereto: (by PROB. 3.) so is D, the fourth Proportional sought: for,

As  $A : B :: C : D$  r.

(Or)

$$\text{As } 8 : 12 :: 16 : 24.$$

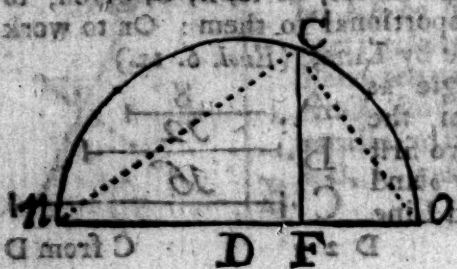
That the young Learner  
may the more readily ap-  
prehend how to draw a  
that is very short, I have  
le Operation of this last Pro-  
compared with Problem III.  
I sufficiently instruct him in

## P R O B L E M   X V I I .

Two Lines A and B, given, to find C a mean Proportional between them. (*Euc.* 2. 14.)



**Note.** A mean Proportional between any two Lines or Numbers, is that which bears the same Proportion to a third Term that the first bears to it.



Join A and B in  
a Right-Line as  
 $n, o$ ; on the mid-  
dle whereof, as at  
D, describe the  
Arch  $n, C, o$ ; and  
from F the Point  
of

of Contact, or meeting of the Lines A and B, erect the Perpendicular FC; which shall be the mean proportional Line required.

For  $EC \times FC = A \times B$ .

That is, the Square of any mean proportional Line, is always equal to the Rectangle or Product of the two Extrems.

### PROBLEM XVIII.

To describe a Circle that shall pass through any three Points, not lying in a Right-Line, as A B C. Join the Points BA and BC with Right-Lines, then (by PROB. 12.) bisect or divide those Lines in the Middle, and continue to draw the bisecting Lines till they meet each other: so shall the Point of Intersection D give you the Centre of the Circle required.

Hence 'tis easy to find the Centre of any given Circle, if three Points be taken any where in the Circumference.

Or, by having a Segment or part of any Circle, to compleat or describe the whole.



### PROBLEM XIX.

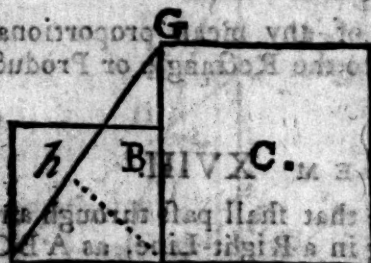
To find two Right-Lines which shall bear such Proportion to each other as two given Squares, viz. B and C.

(Each 6 + 4 = 8.)  
 12 + 2 = 14  
 D 3 Continuing



Continue  $DE$ , the Side of the greater Square, until it be equal to  $FE$ , the Sum of the Sides of both the given Squares  $B$  and  $C$ ; then draw the

Line  $FG$ , subtending the Right  $\angle D$ ; from which let fall the Perpendicular  $Dh$  cutting  $FG$ , in the Point  $h$ ; so is  $Fh$  and  $hG$ , two Lines in like Proportion to each other, as the given Squares  $B$  and  $C$ , as was required.



This Problem might have been otherwise wrought, by finding a third Proportional between the two Sides  $FD$  and  $DE$  (by Prop. 13.) which should have like Proportion to either of them, as the given Squares.

And thus may you at any time find two Lines, bearing such Proportion to each other as any two given Figures whatever, reducing them first into Squares, as you are hereafter directed (by Prop. 26.) and then proceeding by either of the fore-mentioned Ways.

### PROBLEM XX.

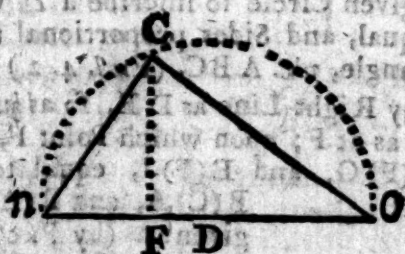
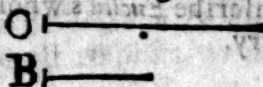
Two Right-Lines  $O$  and  $B$ , bearing any Proportion to each other, being given, to make two like Figures which shall be in the same Proportion one to the other.

(Euch. 6 + 4. 8.)

This Problem is in all Respects perform'd the same with Prob. XVII. foregoing, as the annexed Scheme will sufficiently declare: In which the two Lines  $C$ , and  $C$ , (being found in the same manner as

is

is there shewn) represent the Sides of two equilateral  $\Delta$ 's, or of two Squares, or other like Figures: or the Diameters of two Circles bearing such Proportion to each other, as the given Lines O and B; as was demanded.

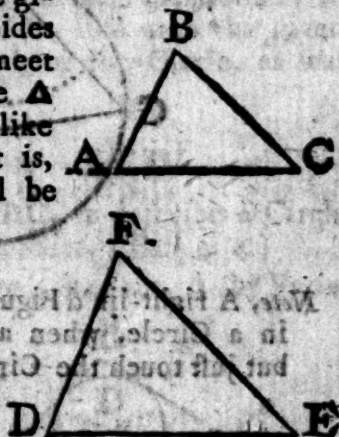


PROBLEM XXI.

Upon a Right-Line, as A C, given, to describe a Triangle similar, or in all respects like to another  $\Delta$ . (*Eucl. I. 37.*)

Make the  $\angle$ 's A and C (by *PROB. 8.*) equal to the  $\angle$ 's D and E, in the given  $\Delta$ : and draw the Sides A B, and C B, until they meet each other: so shall the  $\Delta$  A B C, be in all respects like unto the  $\Delta$  D E; that is, the  $\angle$ 's of the one will be equal to the  $\angle$ 's of the other, and their Sides alike proportional: which was, &c. (See *Theor. 16.*)

This Proposition is of so great Use in the Mathematicks, that it may



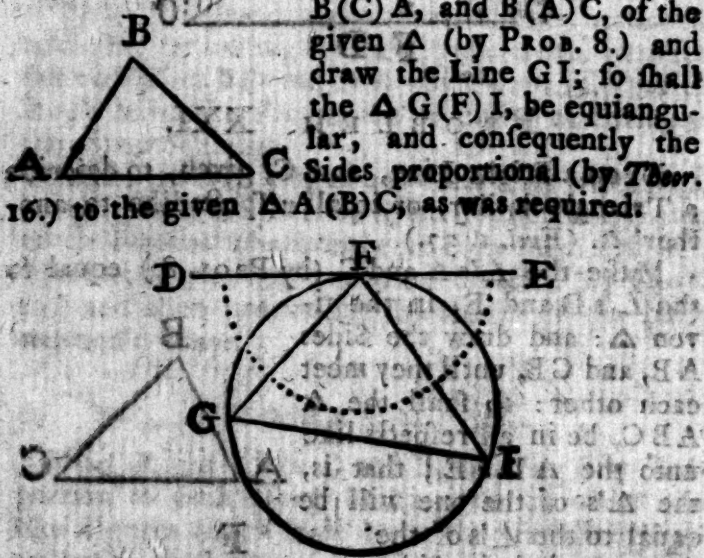
pass

pass for a most universal Principle in taking all manner of Dimensions; and were one but to unfold all the Uses of it; it would be necessary to transcribe *Euclid's* whole first Book of practical Geometry.

# PROBLEM XXII.

In any given Circle to inscribe a  $\Delta$  whose  $\angle$ 's shall be equal, and Sides proportional to another given Triangle, viz.  $\Delta ABC$ , (*Eucl.* 4. 2.)

Draw any Right-Line as  $DE$ , so as just to touch the Circle as at  $F$ ; upon which Point  $F$ , make the Angles  $D(F)G$ , and  $E(F)I$ , equal to the  $\angle$ 's  $B(C)A$ , and  $B(A)C$ , of the given  $\Delta$  (by *Prop.* 8.) and draw the Line  $GI$ ; so shall the  $\Delta G(F)I$ , be equiangular, and consequently the Sides proportional (by *Theor.* 16.) to the given  $\Delta A(B)C$ , as was required.



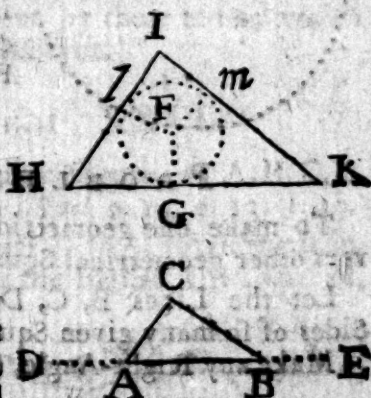
*Note*, A right-lin'd Figure is said to be inscribed in a Circle, when all its angular Points do but just touch the Circle's Periphery.

PROBLEM

PROBLEM XXIII.

About a Circle given, to describe a  $\Delta$  which shall be similar, or alike in all respects, to another  $\Delta$ ; as  $ABC$ . (*Eucl. 4. 3.*)

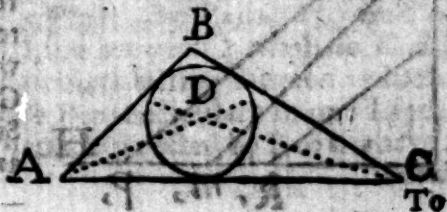
Continue the Base  $AB$ , of the given  $\Delta$ , both ways to any convenient Length, viz. to  $D$  and  $E$ , making the two outward Angles  $D(A)C$  and  $C(B)E$ : then from the Centre  $F$  of the given Circle, draw any Radius, as  $F, G$ , and make the  $\angle$ 's  $IFG$ , and  $mFG$ , equal to the  $\angle$ 's  $D(A)C$ , and  $C(B)E$ , of the given  $\Delta$ ; (by Prop. 8.) and through the Points  $I, m$ , and  $G$ , draw Strain Lines at Right  $\angle$ 's with the several Radius's  $IF, mF$ , and  $GF$ , till they intersect or meet each other; so shall you have the  $\Delta HIK$ , like unto the given  $\Delta$ ; and also described about the Circle, as was demanded.



PROBLEM XXIV.

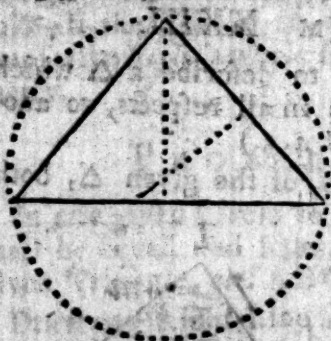
In any given  $\Delta$ , as  $ABC$ , to describe a Circle that shall just touch all its Sides. (*Eucl. 4. 4.*)

Bisect or divide any two  $\angle$ 's of the  $\Delta$  (by Prop. 6.) and where those bisecting Lines meet as at  $D$ , will be the Centre of the Circle required.





To describe a Circle about a given  $\Delta$ , is performed in the very same manner as PROB. 22. viz. by dividing any two Sides of the  $\Delta$  in the middle; and where those Lines meet, each other is the Centre of the Circle required. (See the Figure.)

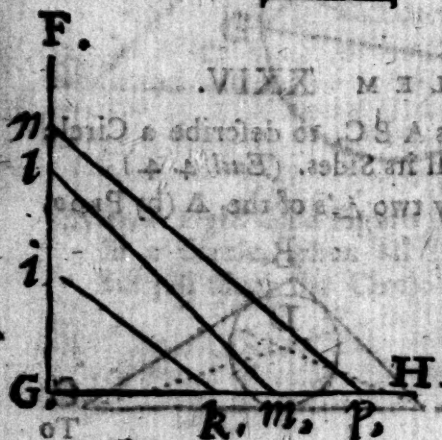
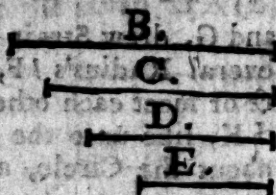


PROBLEM XXV.

To make one geometrical Square equal to divers other geometrical Squares. (*Eucl.* 1. 47. 6. 3 $\kappa$ .)

Let the Lines B, C, D, and E, represent the Sides of so many given Squares.

Make any Right-Angle, (by PROB. 5.) as F, G, H;



then beginning first with the least Side E, place the same from G to i; and also the Line D from G to k; and draw  $ik$ , whose Square shall be equal to the Sum of both the Squares of E and D. Then take the Line  $ik$ , and place it from G to l; and the Side C from G to m, and draw  $lm$ ; which place from G to n; and also the Side B from G to p; and draw  $np$ , which shall

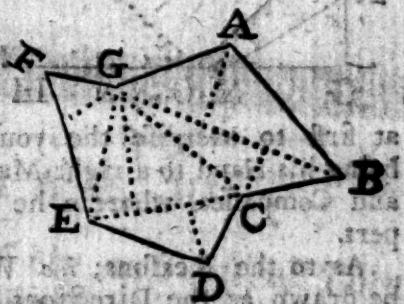
shall the Square made of the last Line, viz.  $ap$ , (by PROB. 11.) be the Sum of all the given Squares represented by their Sides  $B, C, D, E$ ; which was, &c. (See Theor. 18.) And thus by proceeding after the same manner, may any Number of Squares whatever be added together.

Hence also may one Circle be made equal to as many Circles as shall be required, being added by their Diameters as Squares by their Sides; which the Reader may easily discover.

### PROBLEM XXVI.

To reduce an irregular Figure, as  $A, B, C, D, E, F, G$ , into a Square. (*Eucl. 2. 14. 6. 13. 1. 47.*)

Draw Strait Lines from  $\angle$  to  $\angle$ , dividing it into as many Triangles as the Figure will admit of, (which will be always less by 2 than the Number of Sides of the given Figure) as here 5; tho' your better way will be to order Matters so as to have two  $\Delta$ 's to one and the same

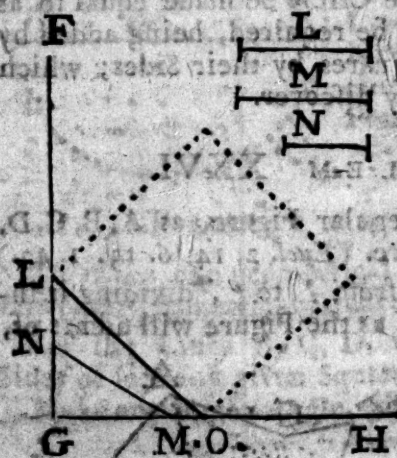


Base, as often as you can, and as you see done in this Scheme, where we have so contrived it, as to have but one single  $\Delta$  in the whole Figure, viz. the  $\Delta F(G)E$ ; the rest fall two upon a Base, as the prick'd Lines in the Figures do plainly demonstrate.

Having gone thus far, what follows is performed after this manner. First, (by PROB. 17.) find a mean Proportional Line between  $\frac{1}{2}$  the Base  $GB$ , and the two Perpendiculars falling thereon; which is done by joining the Perpendiculars in one Line, and proceeding as you are directed by PROB. above.

Work

Work thus until you have gotten a mean Proportional between each Base and the two Perpendiculars falling thereon; and, lastly, between  $\frac{1}{2}$  the Base of the  $\triangle F(G)E$ , and the Perpendicular thereof; so shall you obtain the Right-Lines L, M, N; with which proceeding in all respects the



same as in the last Problem, you will find LO, the Side of a Square, equal to the Figure A, B, C, D, E, F, G, as was required: all which is evident from only a bare Inspection of the Scheme hereunto annexed. (See Theor. 18.)

Thus much will, I think, be sufficient at first to exercise the young Practitioner, and bring his Hand to a right Management of a Ruler and Compasses, wherein he ought to be very expert.

As to the Reasons; *i. e.* Why such Lines must be drawn as the Directions of each Problem require: It is not expected that a young Beginner should at first hand understand them; let him rest satisfied if he be able rightly to perform them, until he hath duly considered the Work of the next Chapter, using himself at the time of reading, to turn back to the Numbers referred to; and then 'tis highly probable he will be capable to see into these Matters.

'Tis not because I don't foresee that by adding this Direction, some may perhaps object I contradict my own Advice, *viz.* That a Learner soon should not

be contented with a bare Knowledge of Things as he passes along, or an Ability only to perform them. I say so still; where there is a Capacity for it, and the Labour to a young Beginner is not too great, and the Advantage thereby gained but small, and not worth the Time and Pains spent about it. Otherwise, (for I would not discourage them) if after they have considered attentively any Proposition, and find they don't understand it thoroughly, it may be passed over until the next Reading, or, at least, till they have gone a little farther, when possibly it may be rendred easy and intelligible from the necessary Connexion and Dependence of these Things one upon another. This is F. Purdie's Advice to those who would understand Geometry; and I think it may be well worth the observing here.



### C H A P. III.

*A Collection of the most useful Theorems in Plain Geometry; whereby the Ground and Reason of the Operations of the last Chapter will be made manifest; with a new and easy Method of demonstrating several of the Theorems contained herein.*

**T**O render the Business of this Chapter as easy and intelligible as possible to the meanest Capacity, I shall first of all lay down some few plain

E

Axioms.

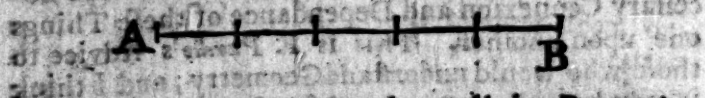


Axioms, or self-evident Truths, to be admitted as the Ground or Foundation of all that I intend to advance herein; which take as follows.



## A X I O M S.

1. **EVERY** Whole is greater than its Parts; *i. e.* the Line A B is greater than any Number of its Parts less than the whole Line.



2. Every Whole is equal to all its Parts; *i. e.* the whole Line A B is equal to all its Parts taken together. The same is to be understood of Superficies and Solids.

3. If equal Quantities be added to, or subtracted from equal Quantities, the Sums or Remainders will be equal.

4. But if to equal Quantities you add, or take away, unequal Parts, the Sums or Remainders will be unequal.

5. If to unequal Quantities you add, or subtract, unequal Quantities, the Sums or Remainders will be equal.

6. Those Lines and Angles which, being placed or laid one upon another, do meet or agree in all their Parts, are equal one to the other.

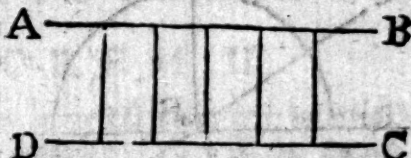
7. Equal Arches of a Circle, or of equal Circles, measure equal Angles.

8. All Right-Angles are equal to one another.

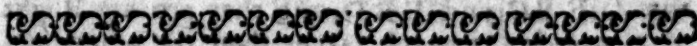
9. If two Lines be parallel, all the Perpendiculars contain'd between them will be equal.

Thus

Thus the Lines AB and DC being parallel, I say, all the Perpendiculars or Strait-Lines drawn between them, are equal.



These Things being premised, I proceed to acquaint the young Geometer with the following necessary Theorems, and their Demonstrations.



### THEOREM I.

If one Right-Line fall upon (or cut) another Right-Line, the  $\angle$ 's made by that Interfection, will be either Two Right  $\angle$ 's, or two  $\angle$ 's equal to Two Right  $\angle$ 's. (*Eucl. 1. 13.*)

Before I proceed to any Demonstrations in this Place, I must again repeat what I have more than once taken notice of in the foregoing Leaves; and I hope the Learner will be admonished hereby to keep it in Memory, especially whilst he is consulting the Demonstration of several of the following Theorems; because I have made it a Preparatory thereunto. The Observation is no more than this: That every Circle is the Measure of Four Right-Angles, a Semi-circle of Two, and a Quadrant (or quarter of a Circle) of one Right-Angle.

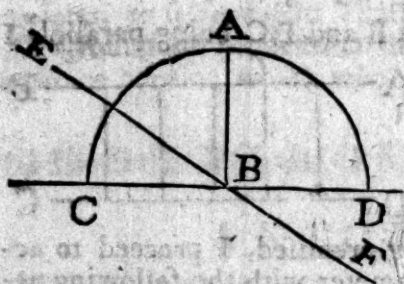
This premised, I proceed to the Demonstration of the Theorem above.

### DEMONSTRATION.

Suppose the Lines AB and CD to meet at the Point B, then it's plain that the  $\angle$ 's AB and C,

E 2

and



and AB and D, are both Right  $\angle$ 's, by the Rule above, because they are each of them the fourth Part of a Circle. But if the two Lines do meet together, as AB and EF, then I think 'tis every whit as plain, that the  $\angle$ 's formed by the Concourse (or meeting) of those Lines, viz. the  $\angle$  EBC and EBD, tho' they are not Right  $\angle$ 's, one being much larger than the other, yet are they both together equal to Two Right  $\angle$ 's, or  $180^\circ$ , since they require a Semi-circle for their Measure.

### COROLLARIES.

1. Hence it follows, that if the  $\angle$  EBC be acute, the  $\angle$  EBD will be obtuse, &c.
2. From hence it will be easy to conceive, that if several Right-Lines stand upon, or meet with any Right-Line in the same Point, all the  $\angle$ 's taken together will be  $= 180^\circ$  Deg.  $= 2$  Right  $\angle$ 's.
3. We may also learn hence, that if at any time one of those  $\angle$ 's be given, the other is easily found, being the Remainder thereof to  $180^\circ$ .

In this Method of Demonstration, there is only this Caution to be observed, That tho' you may make any Circle at pleasure, yet you must remember to measure the  $\angle$ 's always by Arches of the same Circle or Radius; which I mention once for all, in hopes the young Learner will be careful to observe it.



### THEOREM

# THEOREM II.

**I**F two Right-Lines intersect (*viz.* cut or cross) each other, the opposite  $\angle$ 's will be equal (*Eucl.* I. 15.)

## DEMONSTRATION.

Let the two Lines be  $AB$  and  $CD$ , intersecting each other in the Centre  $\odot$ ; upon which let there be drawn the Circle  $ACBD$ , or any other: then I presume it cannot be difficult for any one who has but considered the former Demonstration of the Rule there laid down, to make out this of himself: for if it be remembred, that equal Arches do always measure  $\angle$ 's (as by *Axiom* 7.) there can be no Difficulty in conceiving that the two opposite  $\angle$ 's  $aa$ , and also  $bb$ , are equal, because they are contained under equal Arches; *viz.* the Arches  $CB$  and  $AD$ ,  $CA$  and  $BD$ .

And by the Rule above, the  $\angle$ 's  $a$  and  $b$  on both Sides of the Line  $AB$ , are equal to one another; and also, being taken together, to Two Right  $\angle$ 's, or  $180^\circ$ . The like may be said of the  $\angle$ 's on either Side of the Line  $CD$ ; but this is so plain, that there need no further Proof concerning it.

E 3

COROLLARY



COROLLARY.

Hence 'tis evident, that if two or more Right-Lines intersect (or cut) each other in the same Point, all the  $\angle$ 's made by those Intersections, being taken together, will always be equal to Four Right  $\angle$ 's.

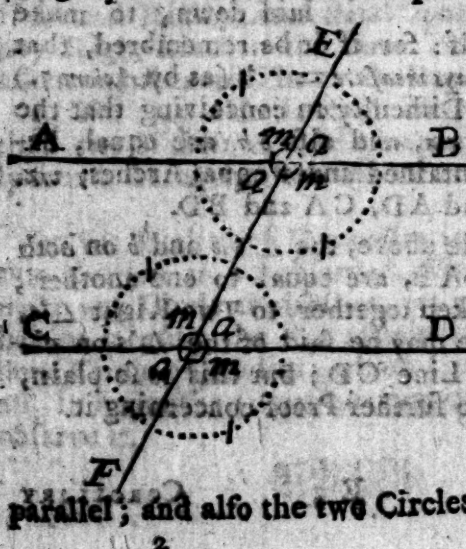
THEOREM III.

**I**F a Right-Line cut (or cross) two or more Parallel-Lines, all the opposite  $\angle$ 's will be equal. (*Eucl.* 1. 29.)

DEMONSTRATION.

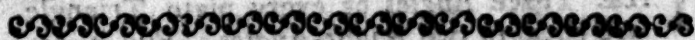
Imagine the Lines AB and CD to be parallel one to another, and that they are cut in the Points  $\odot \odot$  by the Line EF: then upon the Points  $\odot \odot$ ,

with the same Radius, describe the two Prick-Circles, as in the Figure; so will the Demonstration of this Theorem be the same as the last, as is sufficiently evident from only a bare Inspection of the Schemes. For the Lines AB and CD, being parallel; and also the two Circles described thereon,



on, equal to one another; the two opposite Arches of either Circle, mark'd with this Dash (1) will be equal; and consequently the  $\angle$ 's  $m, m$ , and  $m, m$ , contained under them, must be so too. And this is easily tried with a Pair of Compasses, by taking the Distance of any one of those Arches, and applying it to all the rest. The same is to be understood of those Arches which are not mark'd, and of the  $\angle$ 's  $aa$ , and  $aa$ , which are also equal.

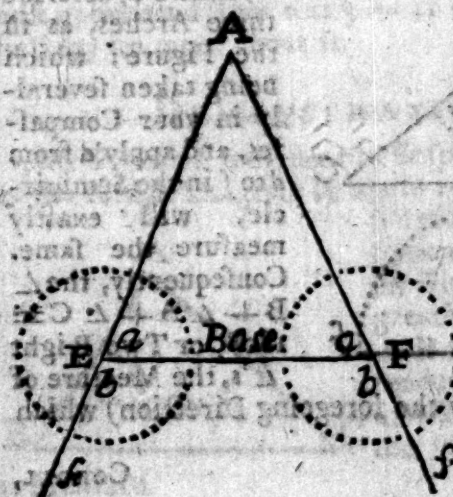
The Converse of this Theorem, is, If a Right-Line falling upon two others, make the opposite  $\angle$ 's equal, those Lines will be parallel. (*Eucl. 1. 28.*)



## THEOREM IV.

**I**N every *Isoceles* Triangle [or that which hath two equal Sides] the  $\angle$ 's above the Base are equal, and also those that are beneath it. (*Eucl. 1. 5.*)

### DEMONSTRATION.



Produce both Sides of the Triangle A (E) F; viz. A E and A F to ff; and on the Points E F, with the same Radius, draw the two prick'd Circles in the Figure: then, if what has been already demonstrated be understood, it must be plain

plain, and I suppose very easy to conceive, that the  $\angle$ 's  $aa$ , above the Base, and also those below it, viz. the  $\angle$ 's  $bb$  are equal, having either of them the same Arches for their Measure; which was to be proved.

On the contrary, we may hence learn, that if two  $\angle$ 's of any Triangle are equal, such  $\Delta$  will be an *Isosceles* Triangle.

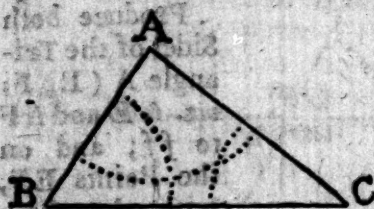
### THEOREM V.

THE Three Angles of every plain Triangle, are equal to two Right-Angles. (*Eucl.* 1. 32.)

And by the same Reason, any two of those  $\angle$ 's must needs be less than Two Right-Angles. (*Eucl.* 1. 17)

#### DEMONSTRATION.

Let  $ABC$  be the  $\Delta$  proposed; draw the Semicircle  $d, e, f$ , and with the same Radius (or opening of the Compasses) setting one Foot in the Points



$A, B$ , and  $C$ , describe three Arches, as in the Figure; which being taken severally in your Compasses, and apply'd from  $d$  to  $f$  in the Semicircle, will exactly measure the same. Consequently, the  $\angle B + \angle A + \angle C \equiv 180^\circ$ , or Two Right  $\angle$ 's, the Measure of

the Semicircle (by the foregoing Direction) which was to be proved.

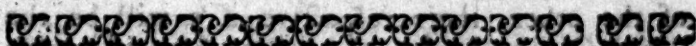
COROLL.

## COROLLARIES.

1. Learn hence, that the Sum of the 3  $\angle$ 's of every plain  $\Delta$  is the same, viz.  $= 180^\circ$ .

2. That if two  $\angle$ 's of one  $\Delta$  be  $=$  two  $\angle$ 's of another  $\Delta$ , their third  $\angle$ 's will also be equal.

3. That if a  $\Delta$  has one Right  $\angle$ , the other two will be acute, and taken together will be  $=$  1 Right  $\angle$ .



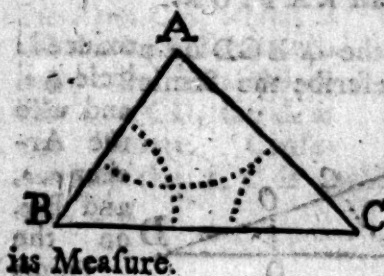
## THEOREM VI.

**I**N every plain  $\Delta$ , the greatest Side subtends (or is oppos'd to) the greater  $\angle$ . (*Eucl* 1. 18.)

Consequently, the greatest  $\angle$  is subtended by the greatest Side; which *Euclid* makes a distinct Proposition by itself; tho' I think it results so naturally from the preceeding Theorem, as to be sufficiently evident without any further notice; for he who understands that twice three makes six, must apprehend that  $6 = 3 + 3$ ; and therefore I shall say no more about it.

## DEMONSTRATION.

This Theorem is evident by Inspection only; for let the former Diagram be again represented, then



its Measure.

it's plain that the  $\angle A$ , subtended by the longest Side, viz.  $BC$ , is greater than either the  $\angle B$  or the  $\angle C$ , subtended by the Sides  $AC$  and  $AB$ , because it has the greater Arch for

COROLL.



## COROLLARY.

Hence it must needs be evident, that equal Sides subtend (*viz.* are opposite to) equal  $\angle$ 's; and contrarily equal  $\angle$ 's are subtended by equal Sides.

Consequently we may also hence learn, That if any two or more Triangles, being compared, have all their Sides equal to one another; that is, if the three Sides of the one be equal to the three Sides of the other, their respective  $\angle$ 's will be the same, *viz.* equal one to the other; since if ever so many of those  $\Delta$ 's were laid one upon another, they would in no part exceed each other, and therefore (*per* AXIOM 6.) are equal.

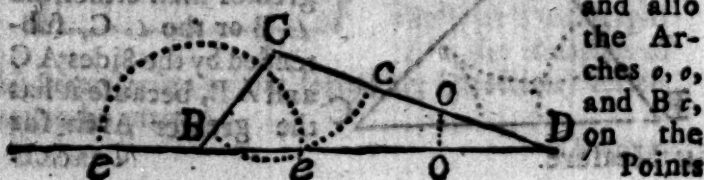
But here note, The Converse of this Theorem does not always hold true, for the  $\angle$ 's of two  $\Delta$ 's may be equal, tho' the subtending Sides of those  $\angle$ 's are unequal.

## THEOREM VII.

**I**N every plain Triangle, if one Side be continued, or produced, the external or outward  $\angle$  thereof will be equal to the two inward and opposite  $\angle$ 's, (*Eucl.* I. 32.)

## DEMONSTRATION.

Let the Side BD of the  $\Delta$  BCD be produced: then on the Point B, describe the Semicircle *e, e*,



Points C and D, (with the same Opening of the Compasses.) Measure the Distance of these two Arches within the  $\Delta$ , and lay down upon the Arch  $e$ , you'll find them to measure it exactly.

Consequently, the  $\angle$  B, without the  $\Delta$ , is =  $\angle C + \angle D$ , within the  $\Delta$ .

## THEOREM VIII.

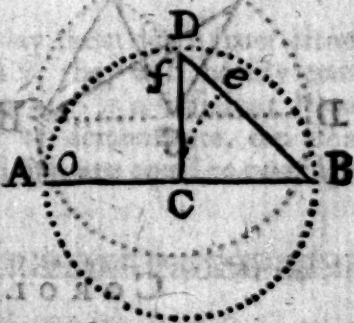
**A**N  $\angle$  at the Centre of a Circle, is always double to the  $\angle$  at the Circumference, when both the  $\angle$ 's stand upon the same Arch. (*Euc. 3. 20.*)

This Proposition is subject to three Varieties; all which, as also their Demonstrations, are very evident, if what has been hitherto mentioned in this Chapter, be rightly apprehended.

### The First Variety.

#### Explanation and Demonstration.

Let the Diameter A B, and the Line D B, be the two Lines which form the  $\angle$  at the Circumference, and the Lines (o) and (f) those which form the  $\angle$  at the Centre C: on the Point B, with the Radius of the Circle A D B, describe the Arch C e, which is just the half of the Arch A D, the Measure of the  $\angle$  A C D, the  $\angle$  at the Centre. Consequently the  $\angle$  C is double to the  $\angle$  B; which was, &c.

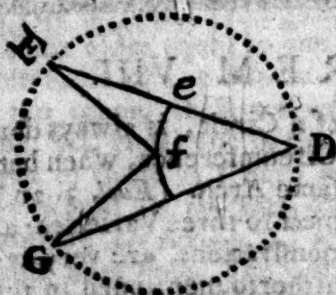


The

## The Second Variety.

### Explanation and Demonstration.

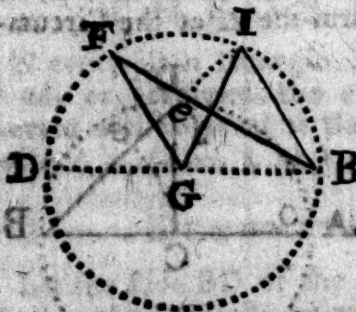
Let the  $\angle E f G$ , be within the  $\angle E D G$ : then if with the Radius of the Circle  $EGD$ , and on the Point  $D$ , you draw the Arch  $ef$ , it will be exactly the  $\frac{1}{2}$  of the Arch of the Circumference  $EG$ : consequently, &c. which was to be demonstrated.



## The Third Variety.

### Explanation and Demonstration.

Again; suppose the  $\angle FGI$ , at the Centre not to be within the  $\angle FBI$ , at the Circumference. If on the Point  $B$  you describe the Arch  $e, I$ , which is the Measure of the  $\angle$  at  $B$ , you'll find it equal to  $\frac{1}{2}$  the Arch  $FI$ , the Measure of the  $\angle G$ : consequently the  $\angle B + B = \angle G$ ; which was, &c.



### COROLLARY.

Hence it's manifest, that all Angles at the Circumference, which stand upon the same Arch of a Circle, or upon equal Circles, are also equal to one another. (*Eucl.* 3. 12.)

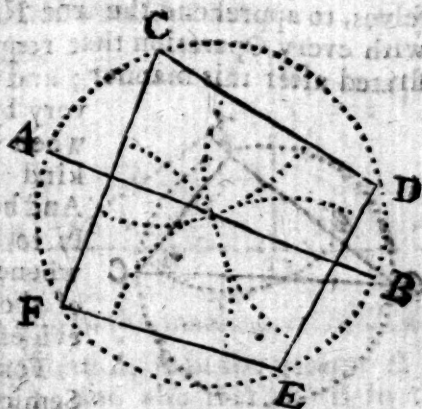
THEOREM

## THEOREM IX.

**I**F any Trapezium (that is, four-sided Figure) be inscribed within a Circle, the two opposite  $\angle$ 's will be equal to two Right  $\angle$ 's, or  $180^\circ$ . (*Euch.* 3. 22.)

## DEMONSTRATION.

Draw the Diameter  $AB$ ; and on every Corner of the Quadrangle, *viz.*  $C, D, E, F$ , (with the same Radius) describe the four prick'd Arches in the Figure; then if you take the measure of any two opposite  $\angle$ 's, which is the prick'd Arch



described thereon, and lay them down from either End of the Diameter  $AB$ , they will exactly compleat the Semicircle; which is a sufficient Proof of what we undertook to demonstrate, *viz.* that the two opposite  $\angle$ 's of, &c. are equal to two right  $\angle$ 's.

## THEOREM X.

**A**N  $\angle$  in a Semicircle is a Right  $\angle$ . (*Euch.* 3. 31.)

F

That

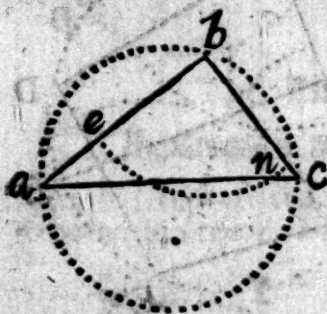


That is, as Mr. *Ward* in his *young Mathematician's Guide* very pertinently expresses it.

"If the Diameter of any Circle be made the Side of a Triangle, and the  $\angle$  opposite to that Side be any where in the Circle's Periphery [or Circumference]" it will be a Right  $\angle$ .

### DEMONSTRATION.

The Matter being thus clearly laid down, and the Method of Demonstration I propose so often repeated, and withal so easy in Practice; I judge it cannot any longer be difficult for any one of themselves, to apprehend the true Nature of proceeding with every Operation that requires to be demonstrated after this manner; and therefore I shall be



very brief for the future, when any thing of that kind happens to occur. And here let it suffice only to shew the Fig. from whence alone the Truth of the last *Theorem* will be evident.

For the Arch  $en = \frac{1}{2}$  the Semicircle  $abc$ ; consequently  $\angle b = 90^\circ$ , the Measure of a Right  $\angle$ , which was to be proved.

### COROLLARIES.

From hence we may easily learn,

1. That an  $\angle$  in a Segment of a Circle less than a Semicircle will be obtuse, i. e. greater than a Right  $\angle$ .

2. That an  $\angle$  in any Segment greater than a Semicircle must needs be acute, viz. less than a Right  $\angle$ .

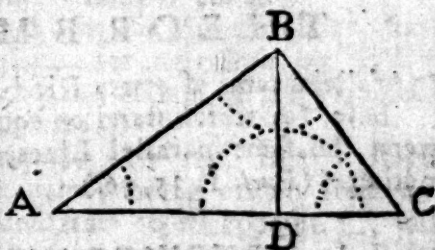
THEO-

## THEOREM XI.

A Perpendicular, drawn from the Right  $\angle$  of any Right-angled Triangle to the opposite Side, shall divide the Triangle into two Triangles similar (or like) to the whole, and also one like unto the other.

## DEMONSTRATION.

In the Right-angled Triangles  $A(B)C$ , let fall the Perpendicular  $BD$ ; then I say that the two  $\Delta$ 's  $A(D)B$  and  $B(D)C$ , are alike

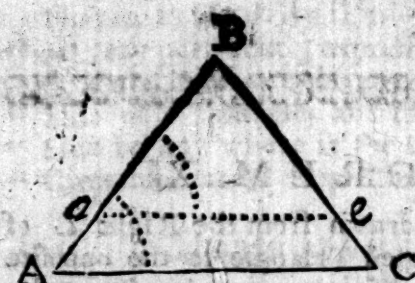


to each other, and also to the whole  $\Delta A(B)C$ . For

1. It's plain, that the  $\angle A$  in the greater  $\Delta = \angle B$  in the lesser  $\Delta$ , and also that  $\angle B$  in the greater  $\Delta = \angle C$  of the lesser  $\Delta$ , as the prick'd Arches in either  $\Delta$  do clearly demonstrate; and as to the  $\angle$  at  $D$ , in both  $\Delta$ 's it is a Right  $\angle$  (*per Theor. 1.*) consequently the two Triangles  $A(D)B$  and  $B(D)C$  are alike. Which was, &c.

And in the same manner may they be proved similar (or alike) to the whole  $\Delta ABC$ : But this is so plain, that it's needless to use any more Words about it.

The like may be affirmed, and proved of a right Line drawn parallel to one of the Sides of any plain  $\Delta$ , *viz.* that it will cut off a  $\Delta$  similar, or alike, to the whole  $\Delta$ .



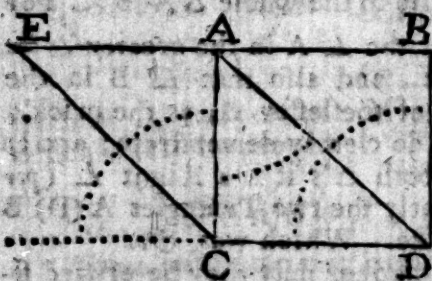
See the Figure in the Margin; whence it may be easily proved, that  $\angle A = \angle a$ , and by Consequence  $\angle C = \angle e$ , and the  $\angle B$  is common to both  $\Delta$ 's.

*Ergo*, the two  $\Delta$ 's  $A(B)C$  and  $a(B)e$  are alike. Q. E. D.

## THEOREM XII.

Parallelograms of every Kind having the same Base, (or which stand on equal Bases, and between the same parallel Lines) are equal to one another. (*Eucl.* I. 35, 36.)

### DEMONSTRATION.



Let  $ABCD$  and  $EACD$  be two Parallelograms, having the same Base  $CD$ , (or equal Bases, which is plainly the same thing) and between the same Parallels,

*viz.*  $EB$  and  $CD$ . Then if the Construction of the Scheme be well consider'd, and the several Parts thereof compar'd together, it must needs be easy to understand the Reason thereof.

For, 1. It's plain that the  $\Delta$ 's  $ECA$ ,  $CAD$  and  $ADB$  are all equal.

But

But  $\triangle CAD + \triangle ADB$  compleat  $\square ABCD$ ,  
and  $\triangle ECA + \triangle CAD$  compleat the  $\square EA$   
CD. Ergo,  $\square ABCD = \square EACD$ . Which  
was, &c.

### COROLLARIES.

Hence we may learn,

1. That all  $\triangle$ 's which stand upon the same Base, or upon equal Bases, and between the same parallel Lines, (that is to say, having the same height) are equal to one another: And also that they are the Halves of those Parallelograms which consist on the same Base (or on equal Bases) and in the same parallel Lines. (*Eucl.* 1. 38. 41.)

Thus the three  $\triangle$ 's in the Figure above, have all of them equal Lines for their Base, *viz*  $EA = AB = CD$ ; and are the Halves of the Parallelograms  $ABCD$  and  $EACD$ .

2. Hence it may be observed, that a Right Line, called a Diagonal, being drawn from any one  $\angle$  of a Parallelogram to its opposite  $\angle$ , will exactly divide the same into two equal Parts, (*Eucl.* 1. 34.) and likewise, that in every Parallelogram the opposite Sides are always equal.

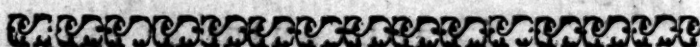
3. From hence also we may apprehend farther, that the Measure of every Right-angled  $\triangle$  is equal to half the Parallelogram, or long Square, made of its Base and Perpendicular.

And generally in all kind of plain  $\triangle$ 's whatsoever, the Area, or superficial Content thereof, is equal to half that Right-angled Parallelogram, (*viz.* long Square) whose Length and Breadth are equal to the Perpendicular, and the Line whereon it falleth.

Whence it follows, to find the Area of any Right-lin'd  $\triangle$ , the Rule is, multiply half the Base by the Perpendicular, or half the Perpendicular by the whole Base, the Product is the Content;



tent. Or multiply the Base by the Perpendicular, and take  $\frac{1}{2}$  of that Product for the Answer: Either of these ways give the true Content.



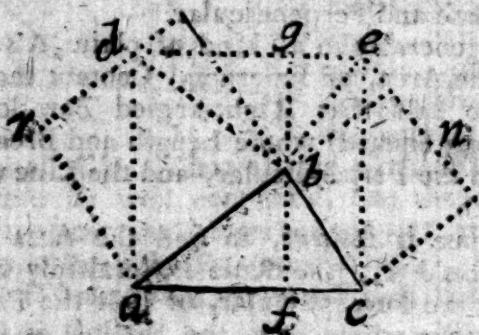
### T H E O R E M XIII.

**I**N any Right-angled  $\Delta$ , the Square made of the Hypotenuse, or Side subtending the Right  $\angle$ , is equal to the Squares of both the other Sides taken together. (*Eucl. 1. 47.*)

This Theorem being the Foundation of a very considerable Part of the Mathematicks, and especially of Trigonometry, which cannot possibly subsist without it, I think it ought to be well consider'd by those who intend any tolerable Proficiency in those Sciences; and therefore I have reserved it for this place, in hopes to make the Demonstration thereof more easily apprehended by a Learner: but especially if what has been said concerning the last Theorem foregoing be rightly digested and understood.

### D E M O N S T R A T I O N.

Let the given  $\Delta$  be  $abc$ ; make a Square of the



Side

Side  $ac$ , as  $deac$ ; do the same of the Sides  $ab$  and  $bc$ , so will you have the three Rectangles as in the Figure; then draw the Line  $fg$  through  $b$ , parallel to  $ec$ ; and also the Lines  $db$  and  $eb$  making the  $\Delta$ 's  $d(b)a$ , and  $e(b)c$ . Then I say,

1. The Square of  $ac$  is equal to the two Rectangles  $df$ , and  $fe$ .

2. The Rectangle  $df$  is double the  $\Delta abd$ , being of the same Base and Altitude, viz. between the same parallel Lines, (*per* Corol. 1. to Theorem the last) and the Rectangle  $fe$  is for the same Reason double the  $\Delta bec$ .

3. But those  $\Delta$ 's being of the same Base and Altitude with the Rectangles  $da$  and  $en$ , (for the Lines  $en$  and  $bc$ , and also  $rd$  and  $ab$  are parallel) will be also equal to one half thereof. Wherefore the Square of  $ac$  is equal to the Sum of the Squares of the Legs  $ab$  and  $bc$ : Which was to be demonstrated.

Among the several ways which have hitherto been contriv'd for demonstrating this famous Theorem, I have not met with any so easy and intelligible for a Learner, as this which I happen'd on in F. Gaston Pardie's Book of the Elements of Geometry, translated into English from the French by Dr. Harris; and tho' I have not set it down exactly in the Doctor's Words, as I find them in the Translation, because I judged them not so fit and plain for a Beginner, yet the Substance is the same.

And because this Theorem, being of such admirable Use in the Mathematicks, cannot be too often inculcated into the Minds of young Beginners, I shall again illustrate the same by Numbers: Thus,

Suppose the Side  $ab = 48$ .

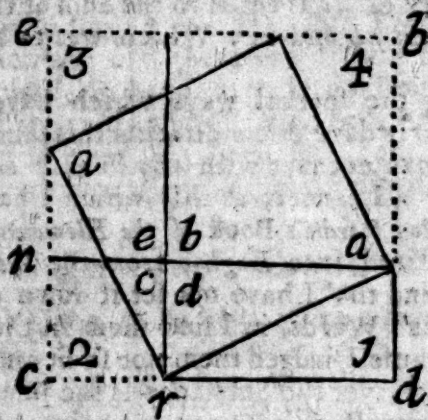
the Side  $bc = 36$ .

And the Hypoth.  $ac = 60$ .

Then

Then will the Square of  $ab$ , viz.  $48 x^d$  into itself  $= 2304$ ; the Square of  $bc$ , viz.  $36 x^d$  by itself  $= 1296$ , the Sum of which two Squares is  $3600$ , equal to the Square of the Hypotenuse, viz.  $60 x^d$  into itself  $= 3600$ ; which proves the Truth of the Theorem as before, &c.

After I had gone thus far, a *learned Friend*, who has more than once oblig'd me in this manner, and whose Name, if I had leave to mention, would do honour to this Treatise, knowing of my Design, was pleas'd of his singular Condescension to send me the following plain and easy Demonstration of this most excellent Theorem; for which I do here return him Thanks, and doubt not but that the Learner will do the same.



In the  $\Delta 1$ ,  $aa$  is the  $\square$  of its Hypotenuse, and  $cc$  and  $bb$  are the  $\square$ 's of the Sides; for  $nc$  and  $ad$  are equal, and also  $ba$  and  $rd$  are equal by Construction: Both which  $\square$ 's, you see, comprehended within the great  $\square cdbd$ ; the  $\Delta$ 's 1, 2, 3, 4, are equal, *per Axiom 6.* hereof. The  $\square dd$  is double the  $\Delta 1$ , *per Coroll. 1. to Theorem 12.* and  
confe-

consequently all the four  $\Delta$ 's, 1, 2, 3, 4, are equal to the two  $\square$ 's  $dd$  and  $ee$ . Now you see that the  $\square$  of the Hypotenuse + the 4  $\Delta$ 's fill the great Square, and so do the  $\square$ 's of the Sides + the 2  $\square$ 's; and take away equal Parts, viz. the 4  $\Delta$ 's on one Side, and the 2  $\square$ 's on the other, and there remain the  $\square$  of the Hypotenuse =  $\square$ 's of the Sides. Q. E. D.

# THEOREM XIV.

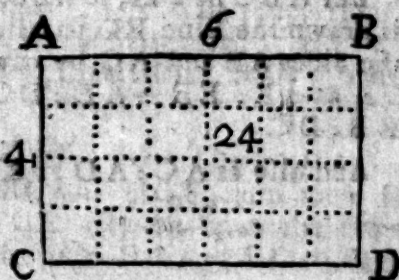
Parallelograms, and also Triangles which are between the same Parallels, (or which have the same Height) are in the same Proportion one to another as their Bases. (Eucl. 6. 1.)

Before I proceed to demonstrate this Theorem, it will be necessary for the Learner to be acquainted with the following Lemma, which serves to prepare the way for the more ready apprehending thereof.

## LEMMA.

A Right Line is said to be multiplied with a Right Line, when either a Square, or other Right-angled Parallelogram is made of the two Lines: That is, the Area or superficial Content of any Rectangle Parallelogram, is equal to the Product of those Numbers which express the Measure of its Sides.

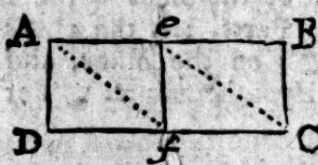
So that if  $AC = 4$  Inches, and  $AB = 6$  Inches; then  $AB \times AC = 6 \times 4 = 24$  square Inches, the Content of the Parallelogram  $ABCD$ .



DEMON-



DEMONSTRATION.



Make the Right-angled Parallelogram ABDC, and draw  $ef$  perpendicular to DC; draw also the prick'd Line  $af$  and  $eC$  constituting the  $\Delta$ 's ADf and  $efC$ .

Then as the Base Df: Base fC ::  $\square ADef$ :  $\square efAC$ ; and  $\therefore \Delta ADf$ :  $\Delta efC$ .

For  $Df \times AD = \square ADef$ . And because  $ef = AD$ , therefore  $fC \times AD = \square efBC$ . But  $Df: fC :: Df \times AD: fC \times AD$ . Ergo,  $\Delta ADf$ :  $\Delta efC :: Df: fC$ , &c. Which was, &c.

THEOREM XV.

**I**F in any Triangle a Right Line be drawn parallel to any Side thereof, it will divide the other two Sides proportionally. (*Eucl. 6. 2.*)

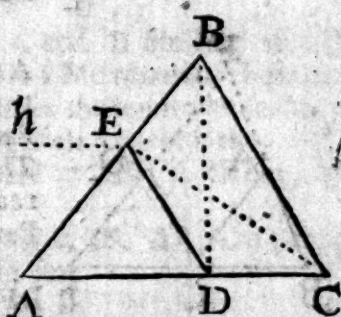
DEMONSTRATION.

Let ABC be a  $\Delta$ , to the Side whereof BC, there is drawn the Line ED parallel thereto; then will the Sides AB and AC be divided proportionally, *i. e.* as AE: EB :: AD: DC; or, as AE: AD :: EB: DC.

And also as AC: AD :: AB: AE.

To

To prove this, draw the prick'd Lines EC and BD forming the  $\Delta$ 's EBD, and ECD, which having the same Base, viz. ED, and being between the same parallel Lines are equal, (*per Corol. 1. to Theor. 12.*)



But the  $\Delta$ 's AED and EDC have the same Height, because they may be placed within the same Parallels, as is evident by drawing  $hE$  thro' the Point E parallel to AC; and consequently have the same Proportion as their Bases, viz. AD and DC, (*per Theor. 14.*) that is,  $\Delta AED : \Delta EDC$  (or its Equal EDB) :: AD : DC

Consequently  $\Delta AED : \Delta EDB :: AE : EB$ . And therefore it must be, as AD : DC :: AE : EB, &c. Q. E. D.

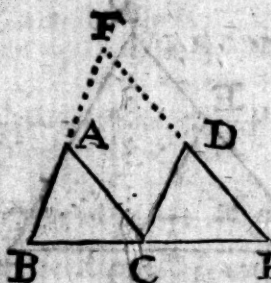
## T H E O R E M XVI.

**I**F two Triangles are alike, their Sides will be proportional. (*Eucl. 6. 4.*)

That is, as Mr. Ward explains it in his *young Mathematician's Guide*; "Those Sides which subtend the equal  $\angle$ 's, as also those Sides which are about the equal  $\angle$ 's, will be proportional to each other. And consequently, if any two  $\Delta$ 's have their Sides proportional, their  $\angle$ 's are equal. (*Eucl. 6. 4, 5, 6, 7.*)

### D E M O N S T R A T I O N.

Let the  $\Delta$ 's be ABC and DCE; join them so as the Bases BC and CE may be upon the same Line, and produce the Sides BA and ED to F.



Then, since the  $\angle$ 's  $ACB$  and  $DEC$  are equal, the Lines  $AC$  and  $FE$  are Parallels, (*per Converse to Theor. 3.*) and by the same reason  $FB$  and  $CD$  are Parallels.

But in the whole  $\Delta$   $BFE$ ,  $FE$  and  $AC$  are parallel; therefore (*per Theor. 15.*)  $AB : AF$  (or which is equal thereto,  $DC$ )  $:: BC : CE$ .

And alternately,  $AB : BC :: DC : CE$ .

So also in the same  $\Delta$ ,  $DC$  and  $AB$  being parallel,  $FD$  or  $AC : DE :: BC : CE$ , (*per Theor. afore.*) And alternately,  $AC : CB :: DE : EC$ , *Q. E. D.*

This is another excellent Theorem, which is so universally useful in all Parts of the Mathematicks, that to unfold all its Uses would require a Volume by itself; for the Business of Trigonometry (both plain and spherical) depends entirely upon it; and therefore it may be truly said, that *Astronomy*, *Dialling*, *Fortification*, *Gunnery*, *Surveying*, *Navigation*, *Opticks*, &c. depend upon a right Application thereof. It's also the Foundation of those excellent Tables of *Logarithmical Sines*, *Tangents*, &c. so valuable for their Use in all the several Branches of the Mathematicks, and of most of the chief Mathematical Instruments now used.

The Converse to this Theorem is evident, *viz.* That those  $\Delta$ 's which have their Sides proportional are Equiangular. (*Eucl. 6. 5.*)

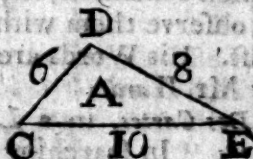
This last Theorem being so considerable in Practice, to make it as plain and easy as possible, I will add another Proof thereof, somewhat different from the former, which perhaps may be more intelligible to some sort of Learners.

Suppose

Suppose the two  $\Delta$ 's A and B similar, or alike to each other, and that the Measures of their Sides are the same as the Figures thereunto annex'd.

Then will the Sides which contain the  $\angle$ 's, and those subtending the  $\angle$ 's, in either  $\Delta$  be proportional, viz.

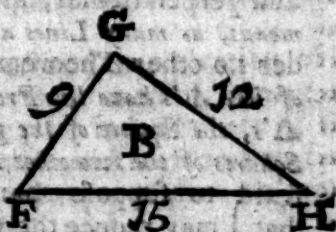
The Sides DC and DE in the  $\Delta$ A, are proportional to GF and GH in the  $\Delta$  B, because their  $\angle$ 's D and G are equal, (*per Converse of this Theorem.*) And therefore it will be as DC: DE :: GF: GH.



And also alternately,

As DC: GF :: DE: GH.

For first, So often as 6 is contain'd in 8, (which is  $1\frac{2}{3}$  Part) so often is 9 contain'd in 12.



In like manner (in the second Proportion) 9 exceeds 6, by just such an aliquot Part thereof, viz.  $\frac{1}{3}$ , as 12 does 8; and therefore the Ratio (or rather Comparifon) of the Sides is equal; and the like of the rest of the Sides which contain  $\angle$ 's.

Also those Sides which subtend  $\angle$ 's, as DC and GF, whose opposite  $\angle$ 's, E and H, are equal; and likewise CE and FH, whose  $\angle$ 's are D and G, are proportional.

For as DC = 6: CE = 10 :: GF = 9: FH = 15. That is, 10 contains the aliquot Parts of 6, so often as 15 contains the aliquot Parts of 12; and therefore, &c.

The same may be said of the other Sides, &c. which I think is very obvious to any one, who is in the least acquainted with these Matters.



"Twill be of great Service to a Learner to understand aright these two *Theorems*, (*viz.* the 13th and last); and also the Nature and Use of parallel and perpendicular Lines, for their admirable Advantage in the Mathematicks, especially in Geometry; as may be farther observed from what Dr. *Pell* writes of *Des Cartes* in his Algebra, pag. 65. which I mention, to put the young Learner in mind to observe them with the more Heed and Carefulness. His Words are these, as I find them quoted by Mr. *Ward*.

*Des Cartes*, in a Letter not yet printed, writes thus: " In searching the Solution of Geometrical Questions, I always make use of Lines parallel and perpendicular, as much as is possible, (*he means, as many Lines as are useful.*) And I consider no other Theorems but these two, [*the Sides of like  $\Delta$ 's have like Proportion.*] And [*in Rectangle  $\Delta$ 's, the Square of the greatest Side is equal to the Squares of the two other Sides*] And I am not afraid to suppose many unknown Quantities, that I may reduce the proposed Question to such Terms, as to depend on no other Theorems but these two.

## THEOREM XVII.

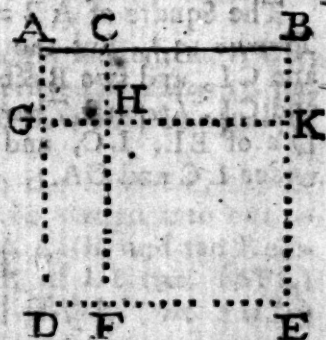
**I**F a Right Line be divided, any how, into two Parts, the Square of the whole Line will be equal to the  $\square$ 's of both the Parts; and two Rectangles contain'd under the same Parts. (*Eucl. 2. 4.*)

### DEMONSTRATION.

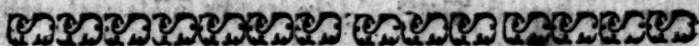
Let AB be divided, by Chance, in the Point C, and its Square ABDE described; from C draw  
CF

CF parallel to AD, and cross it at right  $\angle$ 's, with the Line GK, at the Distance of  $AC = CH$ .

Then it must needs appear to be a Truth, that the Square of the whole Line AB, viz. ABDE, is equal to the Squares made of the Parts CB and CA, viz. HKFE and ACGH, (for HK is equal to CB) and the two Rectangles GF and HB, contained under both the Parts; since being taken together, they fill up no more Space than the Square of the whole Line  $AB = ABDE$ .



I cannot think any further Demonstration of a Matter so evident as this to be needful, except it be to puzzle the Learner; for indeed I am of Opinion, that the too formal and regular Proof of a thing plain in itself, always renders it more obscure and unintelligible.

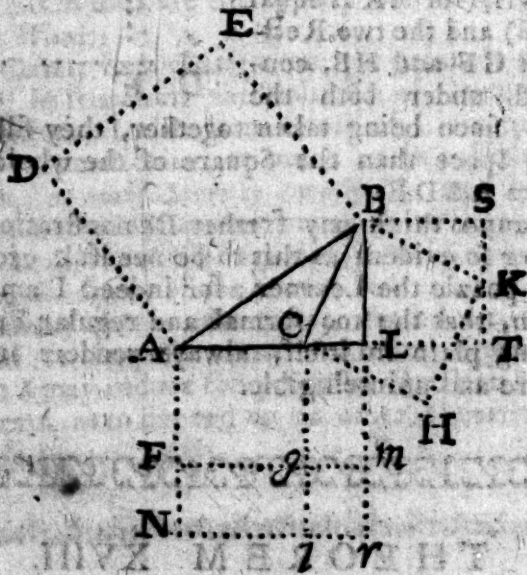


## THEOREM XVIII.

**I**N every Obtuse-angled  $\Delta$ , (as A (C) B) the Square of the Side subtending the Obtuse  $\angle$ , as (C), is greater than the Squares of the other two Sides (AC and CB) by two Rectangles, made of one of the Sides (as AC), and the Segment, or Part, of that Side produced (as CL), until it meet with the Perpendicular (BL) let fall from B upon it. (*Eucl. 2, 12.*)

## DEMONSTRATION.

The Square of  $AB = \square$ 's of  $AL$  and  $LB$ , (*per Theorem 13.*) But the  $\square$  of  $AL = \square$ 's of  $AC$  and  $CL$ , and two Rectangles contain'd under  $AC$  and  $CL$ , (*per last Theor.*) Therefore  $\square$  of  $AB = \square$ 's of  $BL$ ,  $LC$ , and  $CA + 2$   $\square$ 's, contain'd under  $LC$  and  $CA$ .



Instead of the two first  $\square$ 's of BL and LC, made of the Sides of the Right-angled  $\triangle$  BLC, put the  $\square$  of BC, (*viz.* BKCH) which is equal to them both, (*per Theor.* 13.)

Then it's evident, that the  $\square$  of AB, (*viz.* ADEB) is equal to the  $\square$ 's of AC and CB, *viz.* ACFg, and BCHK +  $\square$ 's, CM and gN; and consequently that it's greater than both by 2  $\square$ 's contain'd, &c. Which was to be proved.

**But**

But if this Demonstration be thought too difficult for a young Beginner, he may conceive the Truth of this Proposition by some such easy Method as follows.

Suppose the  $\Delta$  to be Right-angled, as  $ALB$ ; then, as has been said and proved by *Theor. 13.* the  $\square$ 's of the Sides  $AL$ , and  $LB$ , would be equal to the  $\square$  of  $AB$ :

But if from  $B$  be drawn the Line  $BC$ , the  $\Delta$   $ALB$ , by that means, will be parted into two other  $\Delta$ 's, the Obtuse  $\angle$ 'd  $\Delta$   $ACB$ , and the Rectangle  $\Delta$   $CLB$ , and so the  $\square$  of  $LB$  (*viz.*  $BSTL$ ) will be increas'd by the  $\square$  of  $CL$ , (*viz.*  $gm\ lr$ ) for  $gm$  and  $CL$  are equal.

That is, it will be changed into the  $\square$  of the Hypotenuse  $CB$ , *viz.*  $CBHK$ .

But the  $\square$  of  $AL$  (*viz.*  $ALNr$ ) will decrease, and become less by the 2  $\square$ 's  $Ng$  and  $gL$ ; all which is plainly evident by Inspection only. Consequently, &c.

For if to the  $\square$ 's of  $BL$  and  $AL$ , (which are equal to the  $\square$  of  $AB$ ) you add only the  $\square$  of  $CL$  (*viz.*  $gm\ lr$ ), and take away the two  $\square$ 's  $Ng$  and  $gL$ , the  $\square$ 's of  $AC$  and  $CB$  must needs become less than the  $\square$  of  $AB$ . And therefore the  $\square$  of  $AB$  is greater than both the Squares of  $AC$  and  $CB$ , according to the Proposition.

## THEOREM XIX.

IF in any Acute-angled  $\Delta$ , (as  $abc$ ) a Perpendicular (as  $bd$ ) be let fall, the Square of either of the two Sides (as  $ba$ ) is less than the  $\square$ 's of the other Side, and that Side upon which the Perpendicular falls (*viz.*  $bc$  and  $ac$ ) by two  $\square$ 's, made



of the Side  $a c$ , and that Segment, or Part of it, between the Perpendicular and the Side  $b c$ , (*viz.*  $d c$ ). *Eucl.* 2. 13.

This *Theorem* might be demonstrated after the same manner as the last, but I chuse to entertain the Learner with Variety in this place; and the rather, because it will give us an Opportunity of introducing the Method used for subtracting of one Square from another, and producing the Difference between them in a third Square: And also the manner of reducing Parallelograms to Geometrical Squares. With which I shall conclude this Chapter.

### DEMONSTRATION.

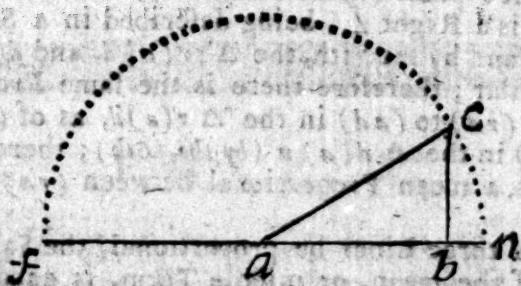
I. By *Problem* 25. of the last Chapter, make a Square equal to both the Squares of the Sides ( $a c$ ) and ( $b c$ ), which let be the Square  $fg ac = \square$  of  $a c + c b$ .

### PROBLEM I.

Then to subtract the  $\square$  of ( $a b$ ) from the  $\square$  ( $fg ac$ ), and to produce the Remainder in a third Square, do thus:

Join the Lines  $f a$  and  $a b$  together in a Right Line, as at ( $a$ ); and from the Centre ( $a$ ) with the Distance ( $a f$ ), describe the Semi-circle  $fn$ ; from ( $b$ ) erect the Perpendicular ( $b c$ ) until it meet with the Circumference, and from ( $a$ ) draw the Line ( $a c$ ). So shall ( $dc$ ) be the Side of a  $\square$ , equal to the Difference





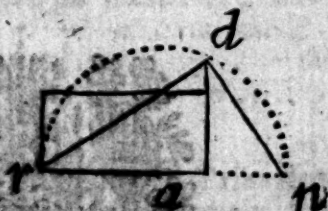
ference between the  $\square$  ( $fgac$ ), and the  $\square$  made of the Line ( $ab$ ): For the  $\square$  of ( $fa$ ) or ( $ac$ ) is equal to the  $\square$ 's of ( $ab$ ) and ( $bc$ ), *per Theor. 13.*

Therefore when the  $\square$  of ( $ab$ ) is taken out of the  $\square$  of ( $fa$ ), the remaining Part will be  $= \square$  of ( $bc$ ), which was requir'd.

## PROBLEM II.

But now, to prove that the  $\square$  of ( $cb$ ) is equal to the double Rectangle made of the Side ( $ac$ ) and ( $dc$ ), viz.  $ra$ , work as follows:

Increase the Side ( $a$ ) of the Rectangle ( $ra$ ) by the Breadth thereof towards ( $n$ ), and on ( $a$ ) describe the Semicircle ( $rn$ ); then if the Side ( $i$ ) be produced until it cut the Circumference, it shall give the Side of a  $\square = \square$  ( $ra$ ), as was to be done; which Line is also equal to ( $bc$ ) in the former Scheme, and shews the Truth of that Operation.



This is, in truth, no other than the Method prescribed for drawing a mean proportional Line to two other given Lines, (in *Prob. 17. of Chap. the last*), as may be easily seen and proved, if you draw the Lines ( $ra$ ) and ( $dn$ ).

For.

For it's plain (*by the 10th hereof*) That the  $\angle r(d)n$  is a Right  $\angle$ , being described in a Semi-circle; and by the 11th, the  $\Delta's r(a)d$ , and  $d(a)n$ , are similar; therefore there is the same Proportion of  $(ra)$  to  $(ad)$  in the  $\Delta r(a)d$ , as of  $(ad)$  to  $(an)$  in the  $\Delta d(a)n$  (*by the 16th*); therefore  $(ad)$  is a mean Proportional between  $(ra)$  and  $an$ .

But if three Lines be proportional, the Square made of the mean, or middle Term, is equal to the Rectangle contain'd under the Extremes; *contra*, (*Eucl. 6. 17.*) consequently the Square of  $(da)$  is equal to the Rectangle  $(ra)$ . *Q. E. D.*

I expect it will be said, that some of these Demonstrations are too mean and mechanick; but I am content it should be thought so, if such as have a Genius this way, and are not able to apprehend those Demonstrations which are more nice and accurate, may be profited hereby; for whose Sake they were chiefly intended.





## C H A P. IV.

*Containing a true Definition of Sines, Tangents, Secants, &c. or those Lines belonging to a Circle, by which the Angles of every Right-lin'd Triangle are measur'd, together with their Construction or Make; shewing how they are deduced from a Circle, and thence transferred to strait Lines; also the Ratio, Reason, or Habitude, which those Lines have in respect of the Diameter of a Circle in natural Numbers.*

## S E C T. I.

**I**N the Mensuration of all Kinds of Triangles, we are first to consider six things, namely, the three Sides and three Angles, of which, as you have heard, (*per Def. 1. Sect. 3. Chap. 1.*) every Triangle is composed. And then by having any three of them given, we may by the Rule of Proportion, (or, as some call it, the Rule of Three) find out a fourth not yet discover'd.

But before this can be done, we must also consider the Analogy or Agreement of the several Parts of a Triangle one to another. That is, in other Words,



Words, we must first find out the Way of reducing crooked Lines to strait, because every crooked Line in a Triangle, as the Measures of the Angles in plain, and the Sides in spherick Triangles are, must first be reduced to a Right Line; which is near enough perform'd by the Definition of the Quantity which Right Lines applied to a Circle have in respect of the Radius or Semidiameter of that Circle. And here it is to be noted, that those Right Lines are usually termed *Chords* (or Subtenses) *Right* and *Versed Sines*, *Co-sines*, *Tangents*, *Co-tangents*, *Secants* and *Co-secants*, according as they respect each other; all which we shall treat of in their Order.

First then, A *Chord* (or Subtense) is a Right Line drawn in a Circle from one Extremity of the Circumference to the other, as the See Fig. 1. Line  $st$ , or  $to$ , which are the Chords of the Arches  $stz$  and  $too$ . Whence we may observe every Diameter is also a Chord Line, and the largest that can be drawn in any Circle whatever.

2. The *right Sine* of an *Arch* is half the Chord of the double Arch: As the right Sine of  $st$  or  $to$  is the Line  $dt$ , it being the half of  $st$ , the Chord of the Arch  $saz$ , equal to twice  $at$ . In like manner the right Sine of  $tc$ , or  $tasm$ , is the right Line  $tb$ , the half of the Chord (or Subtense)  $ta$ . Whence it follows, that the right Sine of an Arch, more or less than a Quadrant, and not greater than a Semicircle, is one and the same; for as the Arch ( $st$ ) is less than a Quadrant by the Arch  $ta$ , so the Arch  $tm$  doth as much exceed a Quadrant, to both which Arches the Line  $tb$  is the right Sine. So that properly speaking, whensoever the right Sine of an Arch is call'd the Sine of the Complement (or Co-sine), we are to understand only the Sine of the Complement of

of an Arch less than a Quadrant, as the right Sine of the Complement ( $ta$ ) is the right Sine of  $tc$ , that is, the Line  $tb$ : Or you may conceive it in other Words; thus; The Co-sine of the Arch  $ct$ , (i.e. the Remainder thereof unto 90 Degrees) is the Arch  $ta$ , the Sine whereof is  $dt$ ; wherefore  $t$  is properly enough said to be the Co-sine, or Sine Complement of the Arch  $tc$ .

3. The right Sine of an Arch is always perpendicular to the Diameter drawn from thence to one Extreme of the given Arch; as the right Sine of  $at$  and  $tc$ , which are the two Lines  $dt$  and  $tb$ , are both of them perpendicular to the Diameters  $an$  and  $mc$ .

4. The right Sine of the Complement, or the Co-sine, of every Arch is equal to that Part, or Segment, of the Diameter which is intercepted, or lieth, between the right Sine and the Centre of the Circle; as  $dt$ , the Co-sine of the Arch  $tc$ , is equal to  $tb$ ; so also the Co-sine of  $at$ , which is the right Line  $tb$ , is equal to  $dt$ . And here it may be useful for the Learner to observe; that the Sine of 90 Degrees, and the Radius, or Semidiameter, of a Circle are all one; for the Sine of an Arch equal to a Quadrant, must be the Radius itself; so the Sine of the Arch  $ac$  is the Semidiameter  $ar$ .

5. The Versed Sine is that Part of the Diameter which lieth between the right Sine and the Circumference; as the Versed Sine of the Arch  $tc$ , is the Line  $cb$ ; and the Versed Sine of  $at$ , is  $ad$ .

6. The Versed Sine of an Arch greater than a Quadrant, is the Line  $d, r, n$ , the Versed Sine of  $tcn$ .

7. The Versed Sine of an Arch less than a Quadrant is  $ad$ , the Versed Sine of  $at$ ; as before.

8. A *Tangent* is a Line so called, for that it toucheth the Circumference, and is always drawn perpendicular to the Diameter, and that End of the Arch which cutteth the same; as the *Tangent* of the Arch  $tc$ , is the right Line  $bc$ .

9. The *Secant* is a right Line drawn from the Centre of the Circle, and the other End of the Arch, until it meet with the *Tangent*; as the *Secant* of the aforesaid Arch  $tc$ , and of the Angle at  $c$  is the Line  $bc$ .

10. The *Difference* of an Arch from a *Quadrant*, or 90 Degrees, whether it be more or less, is called the *Complement* of that Arch; so  $at$  is the *Complement* of the Arch  $tc$ , and also of  $tam$ ; and  $dt$  the *Sine* of that *Complement*,  $ab$  the *Tangent*, and  $cl$  the *Secant* thereof; all which, for brevity's sake, we call the *Co-sine*, *Co-tangent*, and *Co-secant* of that Arch. But,

11. The *Difference* of an Arch from a *Semicircle*, or 180 Degrees, should be called its *Supplement*, and not the *Complement* thereof, as it usually is; so the Arch  $tc$  is the *Supplement* of the Arch  $mat$  to a *Semicircle*.

*Note*, The *Complements* of *Angles* are the same as the *Complements* of *Arches*.

12. The *Measure* of an Angle is the Arch of a Circle described on the angular Point, and included between the two Lines containing the Angle.

Having given the Learner this brief and easy Description of Sines and Tangents, &c. I shall next shew the Fabrick or Geometrical Construction thereof; and then shall define, or express in Numbers, the Relation or Quantity that these Lines have in respect of the Radius, or Semidiameter of a Circle, which is the very Foundation or Ground-work of the Tables of natural Sines, Tangents, &c. so useful in sundry Parts of Mathematics,

ticks, and consequently so necessary to be understood by those, who desire to become tolerable Proficients in those Studies. But first it will be convenient the Learner should here call to mind the foregoing Definition of a Circle, *Chap. I. §. 1.* which we shall briefly repeat as follows.

### DEFINITIONS.

1. The Circumference is that curved Line, which is every-where equally distant from the Centre, or Point on which the Circle is describ'd.

2. The Circumference of every Circle, be it great or small, is by Mathematicians divided into 360 equal Parts, called Degrees; and each of those Degrees are again divided, or suppos'd to be so, into 60 other Parts, called Minutes; so that a Degree is the 360th Part of a Circle: a Semi- (or half) Circle is divided into 180 Degrees, and a Quadrant, or fourth Part of a Circle, contains just 90 Degrees.

3. The Diameter of a Circle is a Right Line drawn thro' the Centre of the Circle, dividing it exactly into two equal Parts.

4. The Semidiameter is one half of the Diameter, and is usually termed the Radius.

### SECT. II.

*The Geometrical Construction of the Lines of Sines, Tangents, &c.*

1. **U**PON a Sheet of fine Pastboard, or any convenient Matter, describe a Semicircle of any Radius; suppose 2 Inches, as *Fig. 2.* in the half Sheet to fold out: Divide it into two equal Parts by the Line AS, drawn perpendicular to the

H

Diame-



Diameter V T. Let each of the Quadrants be divided into 90 Degrees, and number'd from V and S to A, by 10, 20, 30, 40, &c. to 90 Degrees.

2. Draw strait Lines from 10 in one Quadrant, to 10 in the other Quadrant, until you come to 90. The Line SA so divided, shall be a Line of right Sines.

3. Upon the Point T, erect a Perpendicular to L; and from the Centre S, to every 10 Degrees in the Quadrant, draw right Lines till they meet with the Line LT, it shall be a true Tangent Line.

4. Those Lines which issue from the Centre to the Tangent Line LT are called Secants, and may be transferred to the right Line BA, by placing one Foot of your Compasses in the Centre, and describing an Arch from every Degree of the Tangents, to the Line BA; that Line will be a Line of Secants.

5. The Line TV, which is equal to the Diameter of the Circle, or twice Radius ST, is a Line of Versed Sines, and is no other than the Line of *Right Sines* doubled, and number'd from V to T, by 10, 20, 30, &c. to 180 Degrees, as is evident from the Scheme itself.

For want of room, I have drawn the Tangents but to 70°, and the Secants to 60°: But 'tis enough for my purpose, if they had not been extended so far; my Business being only to instruct the Learner how to perform it himself; which is sufficiently done by the Figure itself, and the Directions above.

In order to give the Learner a clear and distinct Idea of these Lines, when they are applied to Practice in the Business of Trigonometry; it will not be amiss if we here observe, that the *Radius*, the *Right Sine*, and the *Right Sine* of the Complement

ment of any Arch, make a plain Right-angled Triangle, as  $rht$ , and the Radius, *Fig. 1.* the Tangent and Secant of the same Arch, make another Right-angled Triangle equiangled, or like to the former, as the Triangle  $rcb$ : And lastly, the Radius, the Tangent Complement of the same Arch, and the Secant of the Complement of that Arch, make another Triangle equiangled, or like to both the former, as the Triangle  $ral$ .

## S E C T. III.

**I** Come now to define, or express in Numbers, the Agreement that these Right Lines have in respect of the Radius, or Semidiameter of the Circle, thus:

Suppose the Semidiameter  $rc$ , to be divided into 1000000 Parts, and that the Arch  $rc$  contains 30 Degrees. Since the Chord or *Fig. 1.* Line which subtends 60 Degrees, is equal to the Semidiameter  $rc$ ;  $tb$ , the Sine of that Arch, will be equal to half  $rc$ , and consequently must contain just 500000 such Parts. Now, by a well known Proposition in Euclid, (*viz.* the 47th, Lib. 1.) which teaches, that in any Right-angled Triangle, the Square of the Hypotenuse is equal to the Squares of both the other Sides; \* we may find the rest, if we

---

\* Note, This Proposition (which is geometrically demonstrated pag. 27. hereof) is of so great Use in the Mathematicks, that its Inventor, Pythagoras, is said to have offered in Sacrifice to the Gods an Hecatomb (i. e. an hundred Oxen) for their assisting him to find it out; supposing it, it seems, above the Power of human Invention.

square  $rt$  (1000000) *i. e.* if we multiply it into it self, it will produce 1000000000000 Parts; from whence subtracting the Square of the Right Sine  $tb$ , *viz.* 25000000, the Remainder (999975000000) will be the Square of  $rb$  and  $dt$ , the Sine of the Complement; and extracting the Square Root of 999975000000, you will have the Line  $rb$ . This done, if you say,

$$As\ rb : tb :: rc : cb,$$

it will give you the Tangent  $cb$ ; then adding together the Squares of  $rc$  and  $cb$ , their Sum (by the said Proposition) will be the Square of  $rb$ ; of which extracting the Square Root, it shall be the Length of the Secant Line  $rb$ .

I shall conclude this Chapter with putting the Learner in mind, to observe two things from what has been said in this and the foregoing Section. And

1. We may hence learn, that the Radius, or Chord of 60 Degrees, the Sine of 90 Degrees, and the Tangent of 45 Degrees, are all equal; for if you take the Length of any one of them between the Compasses, and apply to the rest, it will exactly measure them.

2. Hence also we may perceive the Reason why there are never any Chords, nor but seldom any *Versed Sines* expressed in the Tables, it being indeed a needless thing; for the Chord of any Arch is easily found, by only doubling the right Sine of that Arch; and the Versed Sine of an Arch, if it be less than a Quadrant, is found by subtracting the Cosine of the Arch from the Diameter; or, which is plainly the same thing, that Part of the Diameter which lieth between the right Sine and the Centre of the Circle. But if the Arch be more than a Quadrant, you must add the Cosine of the Arch to the Semidiameter, and it gives the Versed Sine required.

C H A P.



## C H A P. V.

*A brief and clear Explanation of the Logarithms of natural Numbers, and the Table of proportional Parts usually subjoined thereunto; and also of the Table of Sines, Tangents, &c. together with their Use and Operation in the Mathematicks.*

## S E C T. I.

**B**Y whom, or after what manner these most useful and necessary Tables were invented; or what Changes they have since underwent for the better, is not at all material for the Learner to be here acquainted with. Let it suffice in this place to acquaint him with so much of their Use, as shall enable him, not only to understand and perform the Work of this Treatise, but also will be sufficient for almost any kind of mathematical Practice whatsoever.

The best Tables of that Sort, which I have ever seen publish'd, are those in Mr. Taylor's *Thesaurum Mathematica*; which Tables (or those of like Sort) I recommend to the Learner, as the most concise and fit for his Purpose.



P R O P. I.

*Any Number under 1000, or 100000, being given, to find the Logarithm thereof.*

1. **I**F the given Number consist but of one or two Places, its Logarithm and Index are found in the Beginning, or first Side of the Table; as suppose 5 and 21. Look in the Left-hand Column under N, and right against those two Numbers in the next Column are their Logarithms;

As, 5. its Log. = 0.698970

21. its Log. = 1.322219.

After the same manner you will find the

$$\text{Log. of } \left\{ \begin{array}{l} 7 \\ 62 \\ 99 \end{array} \right\} = \left\{ \begin{array}{l} 0.845098 \\ 1.792391 \\ 1.995635. \end{array} \right.$$

The first Figure towards the Left-hand is called the Characteristick, or Index; and in those Tables where it is set down, is always separated from the rest by a Point, or Prick; but in all the late printed Tables it is left out as unnecessary. Its Use is to shew, how many Places of Figures the natural Number consists of, and, in whole or absolute Numbers, is always 1 less in Value than the Number of Places given. Thus, if the whole Number be

|                  |    |
|------------------|----|
| 1                | 0. |
| 10               | 1. |
| 100 its Index is | 2. |
| 1000             | 3. |
| 10000            | 4. |
| Ec.              |    |

Which

Which plainly shews, the Reason why in all Tables they are now left out, is because they are so easily obtained.

2. If the Number whose Log. is sought, consists of three Places of Figures; look in the first Column under N for the Number propounded, and right against, in the very next Column, stands its Logarithm; to which you must prefix 2 for its Index, being 1 less than the Number of Places.

Thus the

$$\text{Logarithm of } \left\{ \begin{array}{l} 241 \\ 567 \\ 999 \end{array} \right\} \text{ is } = \left\{ \begin{array}{l} 2.382017 \\ 2.753583 \\ 2.999565. \end{array} \right.$$

3. But if the given Number consists of 4 Places, then observe the Directions following.

Look first under N for the 3 foremost Places next the Left-hand, as before; then for the last Figure find it on the top; and carrying your Eye downwards in the same Column, until you are opposite to the other 3 Figures under N, you shall there find its Logarithm, to which must be prefix'd its Index 3. So the

$$\text{Logarithm of } \left\{ \begin{array}{l} 2427 \\ 6888 \\ 9999 \end{array} \right\} \text{ is } = \left\{ \begin{array}{l} 3.385069 \\ 3.838093 \\ 3.999957. \end{array} \right.$$

4. But if your Number consist of 5 Places, which is one Place more than the Tables extend to, you must in this Case have recourse to the Table of proportional Parts, usually printed at the End next after the Logarithms; by the Help whereof may be found the Logarithm required: In this manner;

First seek the Log. of the 4 foremost Figures towards the Left-hand, as before: Suppose of this Number

Number 27543, whose Logarithm is 3.439964, the common Difference betwixt which Logarithm, and the next less, found under D, is 158. Which found in the Table of Differences and proportional Parts, right against that 158, and under 3 (the Digit in Units place) stands 47. This added to the Logarithm of 2754, viz. 3.439964, gives 4.440011; which with its proper Index is 4.440011, the Logarithm of 27543 requir'd.

*Other Examples.*

The Logarithm of  $\left\{ \begin{array}{l} 42178 \\ 64829 \\ 26041 \end{array} \right\}$  is  $= \left\{ \begin{array}{l} 4.625086 \\ 4.811769 \\ 4.415657, &c. \end{array} \right.$

Here note, That the Logarithm of any Number is the same, though that Number have ever so many Cyphers added towards the Right-hand; only the Index alters an Unit for every Cypher; as the Log. of 42178, viz. 4.625086, above, is also the Logarithm

Of 4217800, whose Index is 6.

Of 42178000, whose Index is 7.

Of 421780000, &c. whose Index is 8.

~~~~~

P R O P. H.

*To find the Logarithm of a Mix'd Number, also of a Vulgar or Decimal Fraction.*

1. **T**O find the Logarithm of a Mix'd Number, as of 9.567.

Take the Logarithm of 9567, except the Index; and to that Log. prefix an Index according to the Number

Number of Places in the Integral Part of the Number given, as afore directed.

Thus the Log. of 9567 (without the Index) is 980776; and the proper Index for 1 Place is (0). Therefore the Answer is 0.980776; and so for any other Mix'd Numbers. *See more Examples.*

$$\text{The Log. of } \left\{ \begin{array}{l} 369.5 \\ 36.95 \\ 3.695 \end{array} \right\} \text{ is } \left\{ \begin{array}{l} 2.567614 \\ 1.567614 \\ 0.567614 \end{array} \right.$$

Here you see the Logarithms are the same; but the Index is changed according to the Number of Places in the Integral Parts of each Number.

2. To find the Log. of a vulgar Fraction.

Find the Log. of the Numerator and Denominator, and subtract the Log. of the Denominator from the Log. of the Numerator, the Remainder is the Log. required, i. e. of a Decimal equivalent to the vulgar Fraction.

EXAMPLE.

Let the Fraction proposed be  $\frac{3456}{4567}$ .

The Log. of its Numerator is = 3.538574

The Log. of its Denominator is = 3.659631

Which subtracted \* leaves 1.878943

The Log. of .7567 =  $\frac{3456}{4567}$

Now the Reason why the Index is 1, is because there is no Cypher prefix'd to the Decimal; for the

\* N. B. The Rules for adding and subtracting the Indexes being very numerous, and not easily retain'd in mind, I refer the Learner for a full Explication thereof to that curious Piece of Mr. Ed. Hatton's, called his Entire System of Arithmetick.

Index



Index of every Decimal, which hath a significant Figure next the Point, is 1: But if a (o) be set next the Point, the Index is 2; and if 2 (o's), the Index is 3; and so on, always increasing 1 Unit, according as the Fraction removes farther from Unity, as the following little Table shews.

In Decimals the Index of

.1, .22, .342, .671, &c. is 1.

.01, .02, .03, .04, &c. is 2.

.001, .002, .003, &c. — is 3.

.0001, .0002, &c. — is 4.

.00001, &c. — is 5.

Which, by the way, is a sufficient Direction for a Learner to find the Log. of a Decimal; and therefore I shall do no more but give an Example or two, and so proceed. Only it may be needful to observe, first, that the Index of a Decimal is marked with this (—) Mark, put commonly underneath, and called by some a Note of *Negation*. Which serves to show how many Places from the Point the first significant Figure stands.

Examples of finding Logarithms to Decimal Fractions:

$$\text{The Log. of } \left\{ \begin{array}{l} .5 \\ .05 \\ .000579 \end{array} \right\} \text{ is } = \left\{ \begin{array}{l} 11698970. \\ 2.698970. \\ 4.762678. \end{array} \right.$$

*Note*, To find the Log. of a whole Number, which hath a vulgar Fraction annex'd; change it into an improper Fraction, and proceed in all respects as in the last Case. This is so plain, it needs no Example.

P R O P.

P R O P. III.

*Any Logarithm propounded, to find the whole or mix'd Number agreeing thereunto.*

**T**HIS is but the Converse of the foregoing Propositions, and therefore cannot require many Words. Let it then suffice, in this place, to add only one short and general Rule, which is this.

Regard not the Index, but look for the Logarithm among those which would have the greatest Indexes; which having found, take the natural Number standing against it, and point off for Integers according to the Index of the Log. propounded; as in the Examples underneath.

|            |           |                                                          |       |
|------------|-----------|----------------------------------------------------------|-------|
| Logarithms | 3. 987398 | Their Equi-<br>valent,<br>Whole, or<br>Mix'd<br>Numbers, | 9714  |
|            | 2. 932575 |                                                          | 856.2 |
|            | 1. 883945 |                                                          | 76.55 |
|            | 0. 895643 |                                                          | 7.864 |
|            | 1. 089905 |                                                          | .123  |
|            | 2. 954842 |                                                          | .098  |

*Note, If at any time the Logarithm proposed be not found exactly in the Tables, as in many Cases it will so happen, you must then take the nearest Logarithm thereto for the Answer.*

S E C T.

## S E C T. II.

*Of the Use of the Logarithms in Arithmetick.*

**H**ENCE will appear the admirable Advantage of this Contrivance above the natural Numbers, which performs the Work of *Multipli- cation* by *Addition*, *Division* by *Subtraction*; and the Operation of the *Square*, *Cube*, *Biquadrate*, &c. Root; by an easy dividing by 1, 2, 3, 4, &c. as is manifested in the following Instances.

## P R O P. I.

*To multiply one Number by another, as 91 by 36.*

**T**HE Log. of 91 is = 8.959041  
The Log. of 36 is = 1.556303

Which added together gives 3.515344

The Log. of 3276 = to the Product of  $91 \times 36$ .

*Example 2. Multiply 4 by 177.*

To the Log. of 177, which is = 2.247973

Add the Log. of 4, which is = 0.602060

Sum is = 2.850033

The Log. of 708 = to the Product of  $177 \times 4$ .

*Example 3. A mix'd Number by a mix'd, as 9.336 by 12.09.*

To the Log. of 9.336, viz. 0.970161

Add the Log. of 12.09, viz. 1.082426

Sum is = 2.052587

The Log. of 11, 29 = to the Prod. of  $9.336 \times 12.09$ .

These three Examples are sufficient to instruct the Learner how to multiply one Number by another by the Logarithms, and therefore I pass on to give Directions how

P R O P. II.

*To divide one Number by another, as 8463 by 93.*

**F**ROM the Log. of 8463, viz. 3.927524  
Subtract the Log. of 93, viz. 1.968482

Remains ——— 1.959042

The Log. of 91 = to the Quote of  $\frac{8463}{93}$ , i.e. 8463 divided by 93.

*Example 2. A mix'd Number by a mix'd, as 237.3 by 25.32.*

From the Log. of 237.3, viz. 2.375297

Deduct the Log. of 25.32, viz. 1.403464

Rems ——— 0.971833

The Log. of 9.376 = to the Quote of  $\frac{237.3}{25.32}$

P R O P. III.

*To extract the Roots of the Square, Cube, Biquadrat, &c. Powers by Logarithms.*

**T**HESE being the hardest and most difficult Lessons of Arithmetick, are performed by Logarithms with wonderful Ease and Facility, as the following Examples do plainly make appear.

I

*Exam-*



**Example 1.** To extract the *Square Root* of 9801.

**Rule.** Take half the Logarithm of the given Number, that half is the Log. of the Root required.

Thus, half the Logarithm of 9801 = 3.991270  
is = 1.995635

The Log. of 99 the Root sought.

This is proved by a contrary Operation, *viz.* multiplying the Logarithm of 99 by 2, which is the way of squaring any Number by the Logarithms.

So the Log. of 99 = 1.995635

2

$\times^2$  by 2 produces ——— 3.991270  
The Log. of 9801.

**Example 2.** To extract the *Cube Root* of 9261.

**Rule.** Divide the Log. of 9261 = 3.966658 by 3, it produces ——— 1.322219

The Log. of 21 the Root required.

To prove this, multiply ——— 1.322219

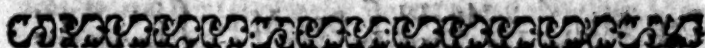
by 3

The Log. of 21 by 3, it will produce — 3.966657

The Log. of 9261 the Cube of 21, *i. e.* the Product of 21  $\times^3$  into itself 3 times.

If any desire the *Biquadrat*, *Sursolid*, &c. Roots of any Number; divide its Log. by 4, 5, &c. This is so easy, that Examples are needless. Only you may note, that if the Index happens to be less than the Number you divide by, you must then take the two first Figures, as in common Division.

S E C T.



## S E C T. III.

*The Explication of the Tables of Logarithmical Sines, Tangents, &c. Shewing how to find the Logarithm of the Sine, Tangent, and Secant; or of the Co-sine, Co-tangent, &c. of any Arch, or Angle of a Triangle.*

**T**HE Method for finding of Numbers by these Tables is very easy; for having found the Degrees on the Top, if the Arch or Angle be less than 45 Degrees; or at the Bottom, if more than 45 Degrees, find the Minutes belonging thereto under M; and opposite to those Minutes, and under the Degrees on the Top, whether *Sine* or *Tangent*, you have the Logarithm answering thereunto; and in the next Column, the *Co-Sine* or *Co-tangent* of that Arch.

*For Example.*

Suppose it were required to find the *Log-Sine*, *Co-Sine*, *Tangent* and *Co-tangent* of 33 Degrees, 45 Minutes.

Turn to 33 Degrees on the Head of the Table, and in the first Column upon the *Right-hand Page* under M, you will find 45 Minutes; opposite to which, and under *Sine* on the Top, is the *Log-Sine* of 33°. 45'. viz. 9.744739. In the next Column the *Co-sine*, the next the *Tangent*, and the next the *Co-tangent* of that Arch; which see hereafter.

Log-sine.

Co-sine.

|           |   |           |   |           |   |             |
|-----------|---|-----------|---|-----------|---|-------------|
| Deg. Min. | { | 9.744739  | { | Deg. Min. | { | 9.919846.   |
| 33 . 45   | { | Log-Tang. | { | 56 . 15   | { | Co-tangent. |
|           |   | 9.824892  |   |           |   | 10.175108.  |

But here note, The Co-sine or Co-tangent of an Arch, being so much as that Arch or Angle wants of 90 Degrees, the Log. of the Co-sine and Co-tangent must have the Number of Degrees and Minutes under M, the Bottom of the Table prefix'd thereto; as here 56°. 15'. which is all one with finding the Log-Sine and Log-tangent of 56°. 15'. For the Log-Sine and Tangent of the *One*, is equal to the Sine-Complement and Tangent-Complement of the *Other*. The same must be observed in all the Degrees and Minutes throughout the Table; which answer, in like manner, those on the Head of the Table to their Complements underneath.

Note also, If the Degrees of any Sine or Tangent exceed 90; you must subtract those Degrees and Minutes from 180, and seek the Sine, Tangent, &c. of the Remainder, which will be the Sine, Tangent, &c. required.

If any desire to find the Logarithmical Secants of Angles, which in many Tables are now left out, (especially in the Tables before recommended) because of the little Use which is made of them in the present Practice, they may be easily obtain'd after this manner; as suppose you want to know the Secant of the Angle 33°. 45'.

Subtract the Sine Complement of that } 20.000000  
 Arch, viz ————— } 9.919846  
 from twice Radius, or 20.000000,

The Remainder is ————— 10.080154  
 the Secant thereof.

And thus may the Secant of any other be found.

And

And here before we conclude this Chapter, it will be convenient to set down the following Method for finding the Arithmetical Complement of a Logarithm, (which is what will make it up 10.000000) as of 9.919846.

Begin with the Figure towards the left-hand, and write down the Complement, or Remainder thereof to 9; and so do with the rest of the Figures till you come to the last, which you must always take from 10.

Thus 9 wants 0, 9 wants 0, 1 wants 8, } 9.919846  
 9 wants 0, 8 wants 1, 4 wants 5, } 0.080154  
 6 wants of 10, 4.

*Note,* The *Co-Arithm.* of any Tangent is the *Cc-*tangent itself.

This *Arithmetical Complement* is very serviceable, when in any Proportion the Radius is not in the first place, for it saves the Labour of Subtraction; as you'll find in many Places of the subsequent Operations in the two next Chapters.

## C H A P. VI

*The Solution of Right-angled plain Triangles, Arithmetically, Geometrical-ly, and Instrumentally perform'd.*

**B**EFORE I proceed to lay down Rules for the Dimensions of Right-lin'd Triangles in general, I suppose it necessary to say something more of the Nature and Affections of such kind of Geometrical Figures than has been hitherto observed, in order to give the Learner a more clear and adequate Idea of those Matters, and to render the



Work of this and the following Chapter plain and intelligible. And here I must first of all desire him to call to remembrance the fifth *Theorem* of *Chap. 3.* which because of the Advantage he may thereby reap herein, I shall here again repeat.

### More Elements of plain Trigonometry.

1. In all plain or Right-lin'd Triangles, the Sum of the three Angles thereof taken together, are equal to two Right ones, or  $180^\circ$ . (See this proved *Pag. 44.*)

From whence may be rationally inferr'd the following easy and undeniable Conclusions.

1. That the Sum of the three Angles of every Triangle is the same.

2. That two Angles of any Triangle being known, the other is also found out; it being the Complement of the rest, i. e. what they want of  $180^\circ$ . or a half Circle.

On the contrary, if you know the Sum of any one Angle, the Quantity of the rest are also known, (as per *Corol. 2. to Theor. 5.*)

3. Hence if any two Angles of one Triangle are equal to two Angles of another Triangle, their third Angles will be also equal.

4. That no Triangle can have above one Right or Obtuse Angle.

5. If in any Triangle one Angle be Right, the other two must be Acute; and taken together, will be equal to one Right Angle.

6. From a Point given, only one Perpendicular can be drawn to the same Line.

7. A Perpendicular is the shortest of all Lines that can be drawn from the same Point to the same Line.

8. In a Right-angled Triangle, the Right Angle is the greatest Angle, and the Side opposite to it the greatest Side. (See *Theor. 6. Chap. 3.*)

9. That

9. That every Angle of an Equilateral Triangle contains just  $60^\circ$ . or a third Part of two right Angles, viz.  $180^\circ$ . (See Def. of an Equilat. Triangle, pag. 8.)

*I cou'd not think it necessary to annex any Scheme to these Corollaries, which are so plain and obvious of themselves, as to need no other Explication but barely to mention them; especially being placed here after all the other Elements of plain Trigonometry, which the Learner is suppos'd so well acquainted with, as to be able to conceive of such easy Matters as these at first Sight.*

2. All Angles whatever concurring (or meeting together) in one Point upon a right Line, being taken together, are equal to two right Angles.

3. In every Triangle two Sides taken together are longer than the third; because a right Line is the nearest Distance between any two Points. This may be thus demonstrated. Suppose a right Line be drawn from the Points A and B, it's then evident, *a*  *b* that no other strait

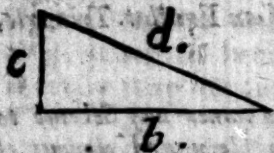
Line can be drawn between the said Points; and consequently if it be crooked, it must be longer than AB (by Def. 4. of Chap. 1.) and such are the two Sides of every Triangle; therefore longer than the third Side. (See the Fig.)

4. The three Sides comprehending the Triangle, some call Legs, other Sides: But when two Sides of a Triangle are considered, the third may be called the Base; tho' that which lies parallel to the Horizon, or next to us, is most properly so termed, as the Line *b*.



Again;

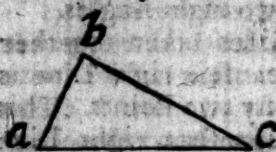
5. Again; Others distinguish the three Sides of a Right-angled Triangle thus: They call the under Side  $b$ , the *Base*; the Perpendicular  $c$ , they call the *Cathetus*; and the longest Side  $d$ , or that which includes the Right Angle, they call the *Hypotenuse*.



But if the Side  $d$  be undermost, the Lines  $c$  and  $b$  are then more fitly call'd Legs, for Distinction's sake.

6. The Angles of all plain Triangles are measured by a Scale of Chords; but the Sides by any Scales of equal Parts, as *Feet, Inches, Yards, &c.*

7. Every Side of a Triangle is call'd the Subtending Side of that Triangle which is opposite to it, as in the Triangle  $a, b, c$ ; the Side  $a, c$  subtends the Angle at  $b$ , and  $a, b$  subtends the Angle  $c$ .



Whence *note*, That the greatest Side always subtends the greatest Angle; the least Side the least Angle; and equal Sides subtend equal Angles.

*Note* also, the two Sides of any Triangle are called the Containing Sides of the Angle contain'd or comprehended between them; as the Sides ( $a$ ) ( $b$ ), and ( $a$ ) ( $c$ ), in the *Fig.* above, are the Containing Sides of the Angle ( $a$ ).

These Things being premis'd, I come next to show, how by having any three of the six Parts pertaining to a plain Triangle, (*the three Angles only excepted*) the rest may easily be obtain'd: But first let me add farther (for the sake of the young Learner) the following Observations.

1. In resolving of plain Triangles, the Angles only being given, the *Reason* or *Proportion* of the Sides

Sides may be found, but not the Sides themselves; it's therefore necessary that one of the Sides be understood.

2. In a Right-angled Triangle, two Terms (*besides the Right Angle*) given are sufficient to give a Third; so that one of the Terms be a Side: But,

3. In Oblique-Triangles, three Things, and one of them a Side, must be given to find a Fourth.

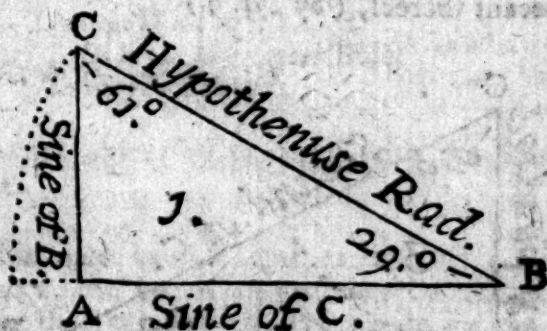
*Of the Solution of the Seven Cases of Right-angled plain Triangles.*

# A X I O M I.

*In every plain Right-angled Triangle, if one of its Sides be made Radius, the other will be either Sines, Tangents, or Secants. That is,*

*As in the first Triangle.*

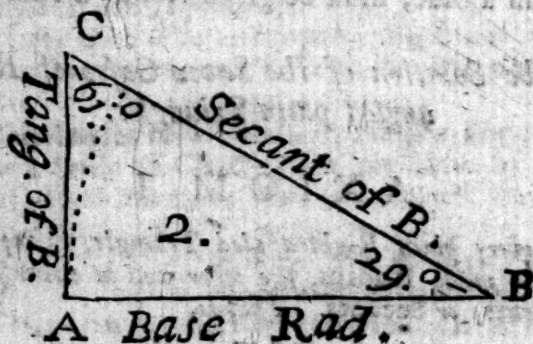
1. If you make B, C, Radius, then will the Sides AC and BA, including the Right Angle, be the Sines of the opposite Angles B and C, (by Def. 3. of Chap. 4.)





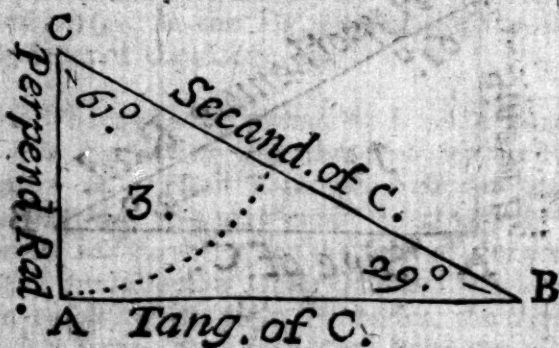
In the Second Triangle.

2. If you make the Side A B Radius, then the other Side A C will be the Tangent of the Angle B, (by Def. 8. of Chap. 4.) and the Hypothenufe BC is the Secant of the said Angle B, (by Def. 9.)



In the Third Triangle.

3. If you make the shortest Side A C Radius, then will the longer Side A B, including the Right Angle, be the Tangent of the greater Acute Angle C, (by Def. 8. above) and the Hypothenufe CB, is the Secant thereof, (by Def. 9.)



Now

Now from the foregoing *Axiom* followeth this *Confectary* or *Conclusion*.

That in all Right-angled plain Triangles, the Angles being given, the Reasons or Proportion of the Sides are also given three ways. And by Consequence,

If one Side, besides the three Angles, be given, the rest of the Sides are obtain'd by a threefold Proportion, according as which Side you make Radius.

And what Proportion soever the Side made Radius hath to the Radius, the very same hath the other Sides to the Sines, Tangents, &c. by them represented; and the contrary.

*Note 1.* That to find a Side, any Side may be made Radius, saying thus:

*As the Word on the given Side,  
Is to the given Side;  
So is the Word on the Side required,  
To the Side required.*

*Note 2.* To find an Angle, one of the given Sides must be made Radius: Then say,

*As one of the Sides given,  
Is to the Word on it;  
So is the other given Side,  
To the Word thereon.*

Always observing to begin with the Side made Radius.

This may be sufficient to instruct a diligent Learner, tho' but of an ordinary Capacity, in ordering the Terms of any Proportion for the Resolution of a right-angled  $\Delta$ . But that I by this first Axiom may make all plain and easy even to

the meanest Understanding, especially in a matter which is of the utmost Consequence to be carefully heeded by a young Practitioner, I shall here add the following Directions for managing the Proportions of all the Cases of right-angled plain  $\Delta$ 's hereafter delivered.

*Direction 1.* If in resolving any of those Questions, the *Hypothenuse* be made *Radius*, and a *Side* demanded, an  $\angle$  must be the first Term in the Proportion, and it must be the  $\angle$  which is opposite to a *Side* given; the *Side* given must be the second Term, and the  $\angle$  opposite to the *Side* demanded the third Term.

*Direct. 2.* But if an  $\angle$  be demanded, a *Side* must be the first Term, and it must be that *Side* that is opposite to an  $\angle$  given; the  $\angle$  given must be the second Term, and the *Side* opposite to the  $\angle$  required the third Term.

*Direct. 3.* If the *Base* be made *Radius*, and the  $\angle$  at the *Base* demanded, then, besides the *Base*, there must be given either the *Hypothenuse*, or *Perpendicular*; and then will the *made Radius* be the first Term in the Proportion, the *true Radius* the second, and the *made Tangent*, if the *Perpendicular* be given; or the *made Secant*, if the *Hypothenuse* be given, will be the third Term.

*Direct. 4.* But if the *made Radius* be demanded, then the *true Tangent* or *Secant* of the  $\angle$  given will be the first Term, the *made Tangent* or *Secant* the second, and the *true Radius* the third Term.

*Direct. 5.* If the *Perpendicular* be made *Radius*, and the  $\angle$  at the *Perpendicular* be demanded; then must either the *Hypothenuse* or *Base* be given, and the *made Radius* must then be the first Term, the *true Radius* the second, and the *made Tangent* or *Secant* the third Term.

*Direct.*

*Direct. 6.* But if the *made Radius* be demanded, the  $\angle$  at the *Perpendicular*, with either the *Base* or *Hypothenufe*, will be given; and then the *true Tangent* or *Secant* must be the first Term, the *made Tangent* or *Secant* the second, and the *true Radius* the third Term. See *Holewell's Trigonometry*, pag. 12.

# CASE I.

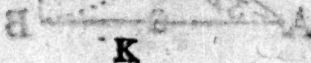
*The two Acute Angles (A) (C), and the Hypothenufe (AC) being given, to find the Base (AB).*

Admit the Triangle given be A, B, C, whose Angle at B is a right Angle (or  $90^\circ$ ), the Angle at C is  $57^\circ. 35'$ , and the Hypothenufe AC is 871.8 Parts. Then will the Angle at A be  $32^\circ. 25'$  it being the Complement of the Angle C to  $90^\circ$ . And this Triangle I shall make use of in solving the seven Cases of right-angled plain Triangles, which I intend to perform three ways, and to shew the several Variations of one and the same Problem. And therefore to solve this first Case,

1. By making the Hypothenufe (AC) Radius.

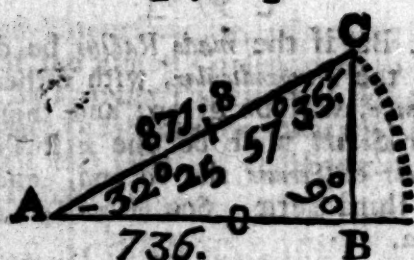
Then will the Analogy or Proportion be (by *Art. 1.* and *Note 1.*) thus:

|                                     |        |          |
|-------------------------------------|--------|----------|
| As Radius, or $90^\circ$ ,          | Parts. | 10.      |
| Is to the Hypothenufe AC, 871.8.    |        | 2.940417 |
| So the $\angle$ C $57^\circ. 35'$ , |        | 9.926431 |
| To the Base AB, 736 Parts.          |        | 2.866848 |



*Observe*





Observe this for a general Rule.

If four Numbers are proportional, to add the Log. of the second and third Terms together, and from their Sum *subtract* the Log. of the first Number; the Remainder is the Log. of the fourth Term, or Number sought, as in the Operation above.

*Note*, That *s.* stands for *Sine*, *c. s.* for *Cosine* (or *Sine-Complement*) *t.* for *Tangent*, *t. c.* for *Tangent-Complement*, (or *Co-tangent*); *sc.* for *Secant*, *Co. Ar.* for *Complement Arithmetical*; and ° ' " set over Figures, denote *Degrees*, *Minutes*, and *Seconds*. And *Note*, Things *given* are marked with a Dash, thus (—), and Things *required*, thus (°).

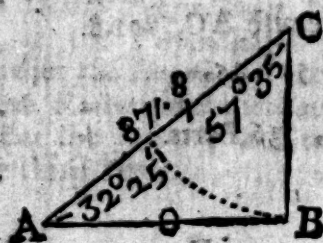
2. By making the Base (A B) Radius,

The Proportion will be,

|                                                      |           |
|------------------------------------------------------|-----------|
| As <i>sc.</i> of the $\angle A$ , $32^{\circ} 25'$ . | 10.073569 |
| To the Hypotenuse AC, 871.8.                         | 2.940417  |
| So Radius $90^{\circ}$ .                             | 10.       |

To the Base A B, 736.

2.866848



3. By

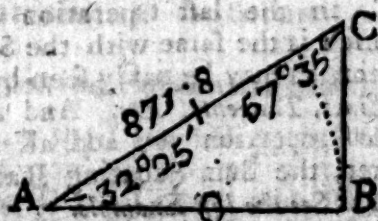
3. By making the Perpendicular Radius.

The Proportion will be,

|                                                              |           |
|--------------------------------------------------------------|-----------|
| As <i>se.</i> $\angle C$ , $57^{\circ} 35'$ . <i>Co. Ar.</i> | 9.729223  |
| To Hypotenuse $AC$ , 871.8.                                  | 2.940417  |
| So <i>s.</i> $\angle C$ , $57^{\circ} 35'$ .                 | 10.197207 |

To the Base  $AB$ , 736.

2.866847



'Tis not of Necessity, that in all the three Operations above, I have put the *Hypotenuse* in the second place. But chiefly for Conveniency in placing the *Log. Figures*; for in any Proportion by *Logarithms*, it matters not how the second and third Terms are disposed of. And therefore, tho' the Nature of Proportion in common Arithmetick requires that the given Terms be so dispos'd, as that the first and third Term be of one Name, and the second with the Term required, (which a young Beginner wou'd do well to observe); yet it is not necessary here.

But here *note*, When *Radius* is not in the Proportion, (as in the last Case it was not), you may more readily perform the Operation by *Addition* only, if instead of the first Term you place its *Arithm. Compl.*; which if it be a *Sine*, may be done as we have shown in Page 89.

So in this Example; 9 wants 0, 9.919846  
 9 wants 0, 1 wants 8, 9 wants 0, 0.080154  
 8 wants 1, 4 wants 5, and 6 *Arith. Comp.*  
 wants 4 of 10.

But if your first Number be either a *Tangent* or *Secant*, you may then take the Co-tangent or Co-sine; for the Tangent-Complement of an Arch is the Co-Arith. thereof, and the Co-sine or Sine-Complement is exactly the Co-Arith. of the Secant of that Arch. So the Co-Arith. of the Secant of the Angle C, in the last Operation above, is 9.729223; which is the same with the Sine Complement thereof, as may be easily seen by a Table of Artificial Sines, Tangents, &c. And therefore, if in any such Proportion you add all the three Terms together, the Sum (abating Radius) will be the Answer. *See the last Example.*

*To perform this Case Instrumentally, by the Lines of Sines, Tangents, and Numbers.*

For the first Proportion:

Which is,

As Rad.  $90^\circ$ . : AC ::  $\angle C$  : BA  
 $871.8$   $57^\circ 35'$   $736$

And therefore,

Extend the Compasses from  $90^\circ$  to  $57^\circ 35'$  in the Line of Sines, the same Extent will reach from 871.8, to 736 in the Line of Numbers.

For which take this general Direction.

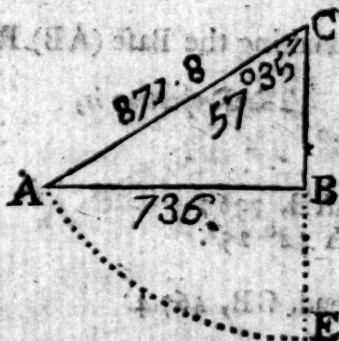
To extend the Compasses always from the first Term to that which is of the same Denomination, whether it be second or third Term, that Extent will reach from the remaining Term to the Answer.

I pre-

I presume it needless to give Instances of the other two Proportions, because the matter is so plain of itself. Only take notice, that where the Term *Secant* is mentioned, there the *Proportion* cannot so easily be wrought but by *Logarithms*. But this the Learner may at any time prevent, by making either of the other Sides Radius. Notwithstanding, if any have a mind to perform such Proportion *instrumentally*, they must account the *Sine* of  $80^\circ$  for a *Secant* of  $10^\circ$ , the *Sine* of  $70^\circ$  for a *Secant* of  $20^\circ$ , &c. and so work by the *Sine* Complement instead of the *Secant*.

*The Geometrical Performance hereof is as follows.*

First, Draw the Line AC, then from any Scale of equal Parts take 871.8; which place from A to C; and upon C as a Centre, with Radius, or  $60^\circ$  of a Line of Chords, describe the Arch AE; upon which set  $57^\circ 35'$  from A to E, and draw the Line CE. Lastly, From A let fall the Line AB perpendicular to BC, so will BA, measured upon your Scale, be found to contain 736 Parts; the thing required.





# CASE II.

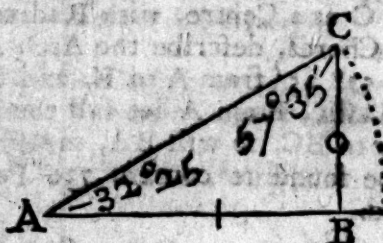
The two Acute  $\angle$ 's (A and C), and the Base (AB) being given, to find the Perpendicular (CB.)

1. By making the Hypotenuse Radius.

The Proportion (by Ax. 1. and Note 1.) is

|                                            |          |
|--------------------------------------------|----------|
| As s. $\angle$ C, $57^{\circ} 35'$ Co. Ar. | 0.073569 |
| To the Log. of the Base AB 736;            | 2.866848 |
| So s. $\angle$ A, $32^{\circ} 25'$ ,       | 9.729223 |

|                           |          |
|---------------------------|----------|
| To the Log. of BC, 467.4. | 2.669640 |
|---------------------------|----------|

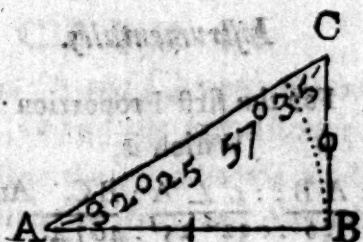


2. By making the Base (AB) Radius.

The Proportion is,

|                                         |          |
|-----------------------------------------|----------|
| As Radius $45^{\circ}$ ,                | 10       |
| To the Base AB, 736;                    | 2.866848 |
| So the s. $\angle$ A $32^{\circ} 25'$ . | 9.802792 |
| To the Perpend. CB, 467.4.              | 2.669640 |

Note,

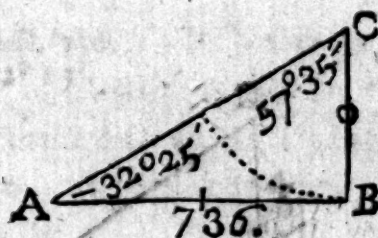


Note, When a *Tangent* is in the Proportion, the Radius then is the *Tangent* of  $45^\circ$ , as in the Operation above: This must be carefully heeded when you work by the Scale.

3. By making the Perpendicular Radius,

*The Proportion is,*

|                                |                 |
|--------------------------------|-----------------|
| As $\angle C$ $57^\circ 35'$   | 10.197207       |
| To Radius $45^\circ$           | 10.             |
| So the Base AB, 736.           | <u>2.866848</u> |
| To the Perpendicular CB, 467.4 | <u>2.669641</u> |



*Instru-*

*Instrumentally.*

For the first Proportion :

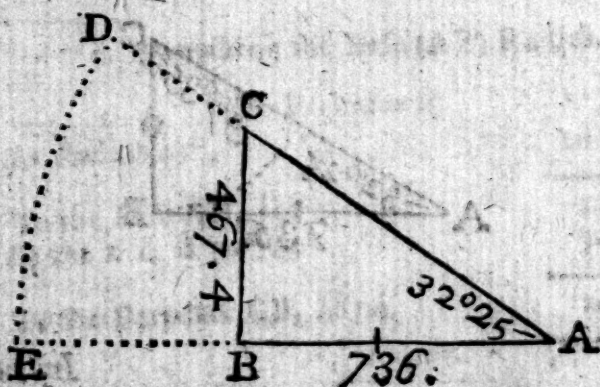
Which is,

As  $s. \angle C : AB :: s. \angle A : BC$ . And therefore,  
 $57^{\circ} 35' : 736 :: 32^{\circ} 25' : 467.4$

if you extend from  $57^{\circ} 35'$ , to  $32^{\circ} 25'$  in the Line of Sines, that Extent will reach in the Line of Numbers from 736 to 467.4 the Answer.

*Geometrically.*

First, Draw the Line  $AB$ , and from any Scale of equal Parts take 736, and set from  $A$  to  $B$ , and from  $B$  erect a Perpendicular; then take  $60^{\circ}$  from a Line of Chords, and placing one Foot of the Compasses in  $A$ , describe the Arch  $ED$ . From the same Line of Chords take  $32^{\circ} 25'$ , and set from  $E$  to  $D$ , and draw the Line  $AC$  till it meet with the Perpendicular  $BC$ ; then if you measure  $BC$  upon the same Scale of equal Parts, it will give 467.4; the Length thereof, as before.



CASE

CASE III.

The two Angles (A and C) with the Base (AB) being given, to find the Hypotenuse (AC.)

1. By making the Hypotenuse Radius,

The Proportion (by *Ar. 1.* and *Note 1.*) is,

As  $\angle C, 57^{\circ} 35'$

9.926431

To the Base AB, 736.

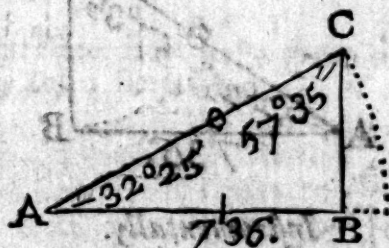
2.866848

So is Radius  $90^{\circ}$ .

10.

To the Hypotenuse AC, 871.8.

2.940417



2. By making the Base Radius,

The Proportion will be,

As the Radius  $90^{\circ}$ .

10.

To the Base AB, 736.

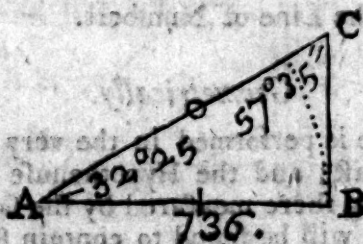
2.866848

So  $\angle A, 32^{\circ} 25'$ .

10.073569

To the Hypotenuse AC, 871.8.

2.940417



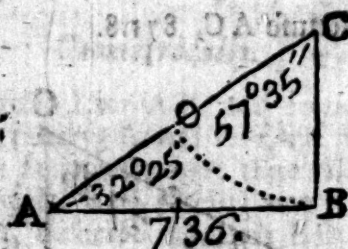
5. By



3. By making the Perpendicular Radius,

*The Proportion is,*

|                                          |                 |
|------------------------------------------|-----------------|
| As $\angle C$ , $57^{\circ} 35'$ Ar. Co. | 9.802792        |
| To Base AB, 736.                         | 2.866848        |
| So $\angle C$ , $57^{\circ} 35'$         | 10.270777       |
| To Hypoth. AC, 871.8.                    | <u>2.940417</u> |



*Instrumentally.*

For the first Proportion:

Which is,

As  $\angle C : AB :: \text{Rad.} : AC$ .

$57^{\circ} 35' : 736 :: 90 : 871.8$

And therefore,

Extend from the Sine of  $57^{\circ} 35'$ , to Radius, or Sine of  $90^{\circ}$ , the same Extent will reach from 736 to 871.8 in the Line of Numbers.

*Geometrically.*

This Case is performed in the very same manner as the last; and the Hypotenuse AC of that Scheme being there measured by the same Scale of equal Parts, will be found to contain 871.8, as by the Proportion above.

CASE

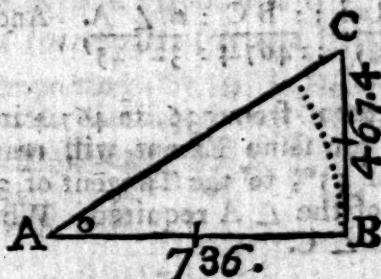
CASE IV.

*The Base (AB) and Perpendicular (BC) being given to find either of the  $\angle$ 's (A and C.)*

1. The Proportion (by Ax. 1. and Note 2.) making the Base AB, Radius, to find the  $\angle$  A, is,

|                                 |          |
|---------------------------------|----------|
| As the Base AB, 736.            | 2.866848 |
| To Radius $45^\circ$ .          | 10.      |
| So the Perpend. BC, 467.4       | 2.669640 |
|                                 | <hr/>    |
| To $\angle$ A, $32^\circ 25'$ . | 9.802792 |
|                                 | <hr/>    |

Which subtracted from  $90^\circ$ , gives  $57^\circ 35'$  the  $\angle$  C, (by Def. 2. hereof.)

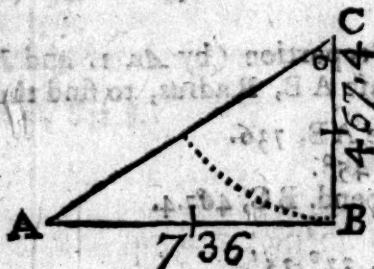


2. The Proportion making the Perpend. Radius to find the  $\angle$  C, is,

|                                 |           |
|---------------------------------|-----------|
| As the Perpend. BC, 467.4.      | 2.669640  |
| To Radius $45^\circ$ .          | 10.       |
| So the Base AB, 736.            | 2.866848  |
|                                 | <hr/>     |
| To $\angle$ C, $57^\circ 35'$ . | 10.197208 |
|                                 | <hr/>     |

Which

Which being taken from  $90^\circ$ , leaves  $32^\circ 25'$  the  $\angle A$ . So that either of these Proportions is sufficient to find both Angles.



*Instrumentally.*

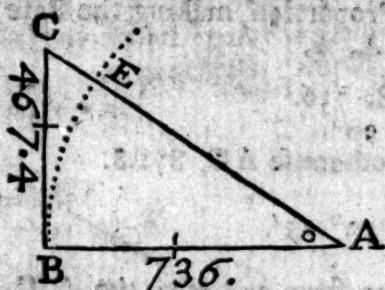
For the first Proportion, which is,

As  $AB : Rad :: BC : \angle A$ . And therefore,  
 $736 : 45 :: 467.4 : 32^\circ 25'$

If you extend from 736, to 467.4 in the Line of Numbers, the same Extent will reach from the Tangent of  $45^\circ$ , to the Tangent of  $32^\circ 25'$ , the Quantity of the  $\angle A$  required. Whose Complement is the  $\angle C$ .

*Geometrically.*

1. Make  $AB = 736$ , and upon B erect the Perpendicular BC.
2. Make  $BC = 467.4$ , and draw the Hypotenuse AC.
3. With 60 Degrees of a Line of Chords draw the Arch BE; then if you measure BE with the same Scale of Chords, you'll find it  $= 32^\circ 25'$  the Angle A; the Remainder whereof to  $90^\circ$  is  $57^\circ 35' = \angle C$ .



I suppose the Learner, by this time, so well acquainted with the several Signs or Characters explain'd and used in this Book, and the Geometrical Protraction of this kind of Triangle, as to need but few Words to apprehend it.

### C A S E V.

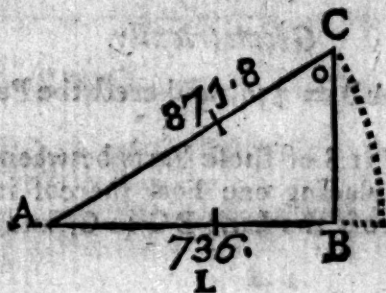
*The Base (A B) and Hypothenufe (A C) given, to find the Angles (A and C.)*

1. The Proportion (by *Ax. 1. Note 2.*) making the Hypothenufe Radius, to find the  $\angle C$ , is,

|                                |          |
|--------------------------------|----------|
| As the Hypothenufe A C, 871.8. | 2.940417 |
| To Radius $90^\circ$ .         | 10       |
| So the Base A B, 736.          | 2.866848 |

|                                   |          |
|-----------------------------------|----------|
| To s. $\angle C$ $57^\circ 35'$ . | 9.926431 |
|-----------------------------------|----------|

The Complement whereof to  $90^\circ$  is the  $\angle A$ .



2. The



2. The Proportion, making the Base Radius, to find the  $\angle A$ , is,

As Base AB, 736.

2.866848

To Radius  $90^\circ$ .

10.

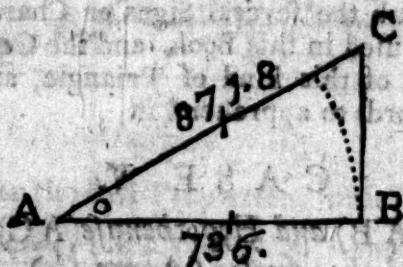
So the Hypotenuse AC, 871.8.

2.940417

To  $\angle A$   $32^\circ 25'$ .

10.073569

Which taken from  $90^\circ$  gives the  $\angle C$ , as before.



*Instrumentally.*

For the first Proportion, which is,

As AC : Rad. :: AB :  $\angle C$ .

871.8 : 90 :: 736 :  $57^\circ 35'$ .

And therefore

Extend from 871.8 to 736 in the Line of Numbers, that Extent will reach from  $90^\circ$  to  $57^\circ 35'$  in the Sines.

*Geometrically.*

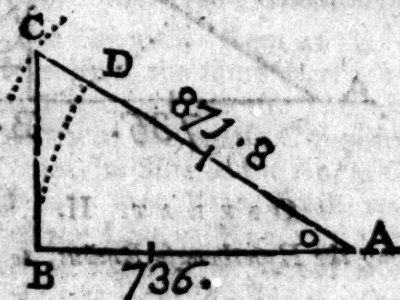
1. Make AB = 736, and erect the Perpendicular BC.

2. Take 871.8 of those Parts between the Compasses, and placing one Foot thereof in A, with the other cross the Line BC in C, and draw the Line AC.

3. With

[ III ]

3. With  $60^\circ$  of Chords describe the Arch BD; which being measured upon the Chords will be  $= 32^\circ 25'$ , the  $\angle A$  required; whose Complement is the Angle C.



CASE VI.

*The Base (AB) and Perpendicular (BC) given, to find the Hypotenuse (AC).*

The resolving of this and the following Case will require two Operations, one to find an Angle, and the other, by help of that Angle, to find the Side required.

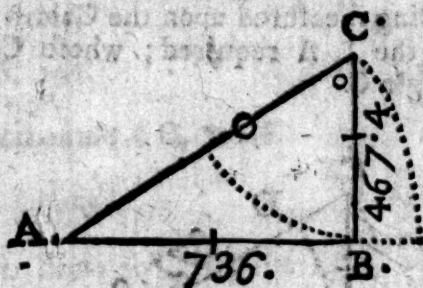
OPERATION I.

*By making the Perpendicular Radius, to find the  $\angle C$ .*

The Proportion (by Ax. 1. Note 2.) is

|                                            |           |
|--------------------------------------------|-----------|
| As the Perpendicular BC, 467.4.            | 2.669640  |
| To Radius $45^\circ$ .                     | 10.       |
| So the Base AB, 736.                       | 2.866848  |
| To 2. of the $\angle C$ , $57^\circ 35'$ . | 10.197208 |

*Note, This is the same with the second Prop. of Case 4.*



OPERAT. II.

Making the Hypothenuſe Radius, to find AC.

The Proportion (by Ax. 1. Note 1.) is

|                                   |          |
|-----------------------------------|----------|
| As $s. \angle C, 57^{\circ} 35'.$ | 9.926431 |
| To Baſe AB, 736.                  | 2.866848 |
| So Radius $90^{\circ}.$           | 10.      |
| To Hypothenuſe AC, 871.8.         | 2.940417 |

Which is the ſame Operation with Prop. 1. of Caſe 3. and may be reſolved (as that Caſe there is) ſeveral ways, as may be ſeen by the following Proportions, (making each Side Radius) which we have contracted into as ſmall Compaſs as poſſible.

1. The Proportions making the Hypothenuſe AC Radius, are,

$$\text{As } \left\{ \begin{array}{l} s. \angle C : AB \\ \text{Or,} \\ s. \angle A : BC \end{array} \right\} :: \text{Rad} : \text{Hypoth. AC.} \quad \underline{871.8}$$

2. Making the Baſe AB Radius.

$$\text{As } \left\{ \begin{array}{l} \text{Radius} : AB \\ \text{Or,} \\ s. \angle A : BC \end{array} \right\} :: s. \angle C : \text{Hypoth. AC.} \quad \underline{871.8}$$

3. Ma-

3. Making the Perpend. BC Radius.

As  $\left\{ \begin{array}{l} 1. \angle C : AB \\ \text{Or,} \\ \text{Radius : BC} \end{array} \right\} :: \text{fe. } \angle C : \text{Hypoth. AC.}$   
871.8

To perform this Case *Instrumentally*.

1. Extend the Compasses from 467.4 to 736 in the Line of Numbers, the same Extent will reach from the Tangent of  $45^\circ$  to the Tangent of  $57^\circ 35'$ , the Quantity of the Angle C required.

2. Extend from the Sine of the Angle C  $57^\circ 35'$  to the Sine of  $90^\circ$ , that Extent will reach from 736 to 871.8 in the Line of Numbers.

The Geometrical Protraction hereof is the same as of Case 4. where, beside finding the Quantity of the  $\angle$ 's A and C, if you measure AC, the Hypothenuse, you'll find it to contain 871.8.

C A S E VII.

*The Base (AB) and Hypothenuse (AC) given  
to find the Perpendicular.*

The two Operations making the Hypothenuse  
Radius are as follow :

|                                                         |          |
|---------------------------------------------------------|----------|
| As Log. Hypoth. AC, 871.8.                              | 2.940417 |
| To Radius $90^\circ$ .                                  | 10.      |
| So Log. Base AB, 736.                                   | 2.866848 |
| To $\angle$ C, $57^\circ 35'$ .                         | 9.926431 |
| Whose Comp. to $90^\circ$ is $32^\circ 25'$ $\angle$ A. |          |



Then say,

As Radius  $90^\circ$ .

IO.

To Log. Hypoth. AC, 871.8.

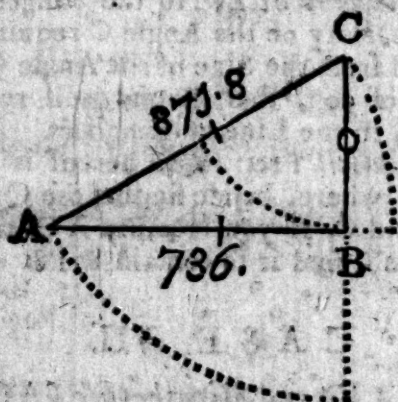
2.940417

So  $\angle A$   $32^\circ 25'$ .

9.729223

To Log. Perpend. 467.4

2.669640



Having found the two acute Angles A and C by the first Operation, the three several Proportions, making each Side Radius, to find the Perpendicular BC, are the same with those belonging to Case 2. as may be seen by the following Contraction.

1. Making the Hypothenuſe Radius;

$$\text{As, } s: \angle C : AB :: s: \angle A : BC;$$

$$57^\circ 35' : 736 :: 32^\circ 25' : 467.4$$

2. Making the Baſe Radius;

$$\text{As, Radius} : AB :: s: \angle A : BC.$$

$$45^\circ : 736 :: 32^\circ 25' : 467.4$$

3. Ma

3. Making the Perpendicular Radius;

As,  $\angle C : \text{Radius} :: AB : BC.$   
 $57^{\circ} 35' : 45^{\circ} :: 736 : 467.4$

This and the last Case may be otherwise resolved by the 13th *Theorem* of *Chap. 3.* hereof, which proves that the  $\square$  of the Hypotenuse of a  $\triangle$  Triangle, is  $=$  to the Sum of the  $\square$ 's of the other Sides; and therefore, when both the Sides are given to find the Hypotenuse (as in *Case 6*)

*The Rule by Logarithms is,*

From double the Log. of the greater Leg, subtract the Log. of the other, and add the absolute Number belonging thereto to the lesser Leg.; then is half the Sum of the Logarithms of that Sum, and lesser Leg. the Log. of the Hypotenuse required.

*Example.* The Side AB and BC given to find AC.

The Log. of AB 736, the greater Side is, 2.866848

Which  $\times$  by 2, gives the double Log. of AB, 5.733696  
 From which take the Log. of the les. Side BC, 2.669640

The Remainder is — 3.064056

The absolute Number of which is, 1159  
 $467.4 = 2.669640$

Sum of which, and lesser Leg. } 1626.4  
 BC 467.4 is

Whose Log. is — 3.211227

Sum of the two last Logarithms,  $\frac{1}{2}$  5.880867  
 Half whereof is the Log. of AC, 2.940433  
 871.8 the Hypotenuse sought. For

For this last Case the Rule by Logarithms is,

Add the Logarithms of the Sum and Difference of the Hypotenuse and Base; half their Sum is the Log. of the Perpendicular.

*Example.* A B and A C, to find B C.

The Hypotenuse A C, 871.8 }  
The Base A B, 736 } added.

Sum 1607.8 Log. 3.206232

Differ. of A C and A B, 135.8 Log. 2.132899

Their Sum — 5.339131

Half Sum — 2.669565

Whose absolute N° 467.3 is the Perpend. sought.

To perform this seventh Case *instrumentally*.

1. The Extent from 871.8 to 736, in the Line of Numbers, will reach from 90° to 57° 35' in the Line of Sines.

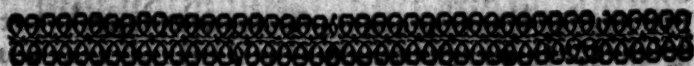
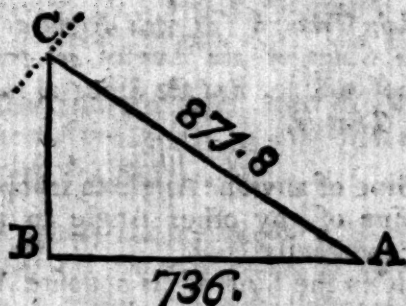
2. The Extent from 90° to 32° 25' on the Sines, will reach from 871.8 to 467.4 on the Line of Numbers.

*Note here, In working Proportions instrumentally by Scale and Compasses, it will sometimes be needful to use Cross-work, i. e. To extend from the first Term in the Line of Sines to the second in the Line of Numbers; or from the first Term in Tangents to the second in Numbers, especially when the Extent is too large for the Compasses. But in most other Cases I think it better to follow the Directions above.*

The Geometrical Performance is as followeth.

Make A B = 736, and draw the Perpendicular B C; then take 871.8 betwixt your Compasses, and placing

placing one Foot thereof in A, with the other cross the Perpendicular in C; so shall BC, measured upon your Scale, be 467.4, the Length of the Perpendicular required.



## C H A P. VII.

### *Of Oblique Right-lined Triangles.*

**H**AVING, in the former Chapter, sufficiently, fully, and clearly explain'd all the Cases of Plain Right-angled Triangles; I shall now, to compleat the whole Doctrine of the Dimension of Plain Triangles, proceed with the same Plainness to treat of the five Cases of Oblique-angled Triangles remaining. But before I lay down particular Rules for the Resolution thereof, I shall here exhibit the several *Axioms* upon which the whole Work depends, leaving their Demonstrations, which I conceive not so easy for a young Beginner to understand, to be learnt from such Authors as have made it their Business to handle these Matters, for the



the sake of the Ingenious, and who have wrote more learnedly thereof.

## A X I O M I.

*In every Plain Triangle (as well Right as Oblique-angled) the Sines of the Angles are proportional to their opposite Side, and the Sides to their opposite Angles: That is,*

As the Sine of any one Angle to its opposite Side, so is the Sine of any other Angle to its opposite Side; and contrarily.

As a Side to the Sine of its opposite Angle, so is any other Side to the Sine of its opposite Angle. So that if the Side of a Triangle be required, put the Sine of the opposite  $\angle$  in the first place. But if an  $\angle$  be required, put the Logarithm of his opposite Side in the first place.

Take special Notice of the foregoing *Axiom*, (which some call the Rule of Opposites) for its extraordinary Usefulness in most Cases of plain Trigonometry; and if rightly understood, will, with a very few Directions beside, render the whole Business thereof easy and intelligible.

This *Axiom* duly consider'd will afford the following Consequencies:

1. That the  $\angle$ 's of any Triangle being given, the Reason of the Sides is also given; and by consequence,

If one Side besides the  $\angle$ 's be given, the other Sides are also given.

2. That any two Sides with an  $\angle$  opposite to one of them being given, the opposite  $\angle$  to the other Side is also given.

A X I O M

## A X I O M II.

*In all plain Triangles; As the Sum of the two Sides, including the  $\angle$  given, is to their Difference; so is the Tangent of the  $\frac{1}{2}$  Sum of the opposite  $\angle$ 's, to the Tangent of  $\frac{1}{2}$  the Difference of the said  $\angle$ 's.*

This Axiom is useful when two Sides and an  $\angle$  between them, are given to find the third Side, and either of the other  $\angle$ 's.

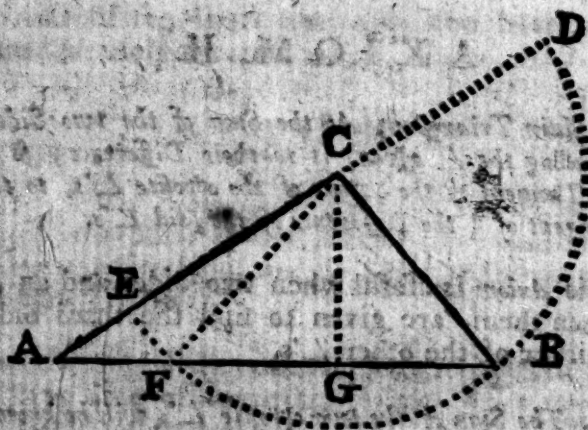
Note, The Sum of the two opposite  $\angle$ 's will be known, being what the given  $\angle$  wants of  $180^\circ$ . and their Difference is found by this Axiom. And by adding the half Sum and  $\frac{1}{2}$  Difference together, you will have the greater  $\angle$  sought: Or by subtracting the half Difference from the half Sum, the Remainder is the Quantity of the lesser  $\angle$  required.

## A X I O M III.

*In all plain Triangles; As the greatest Side is in Proportion to the Sum of the other two Sides; so is the Difference of those Sides to a Part or Segment of the greater; which being taken from the greatest Side, a Perpendicular let fall from the  $\angle$  opposite to the longest Side, shall divide the Remainder in the middle, and the Oblique Triangle into two Right-angled Triangles.*

To illustrate this Axiom more fully and plainly, let us suppose the Triangle A B C to be given, whose greatest Side is A B, and least Side B C.

Upon C as a Centre, at the Distance B C, describe the Semicircle D B E E, and let the Line A C be continued to D; so is A D the Sum of the Sides



Sides  $AC$  and  $CB$ ; for  $CB$  and  $CD$  are equal (by Construction \*). So also are  $CB$  and  $CE$ , and therefore  $AE$  must be the Difference of these Sides.

Hence we may conclude, That as the greatest Side  $AB$  is in Proportion to  $AD$ , the Sum of the other two Sides; so is  $AE$ , the Difference to  $AF$ , a Segment or Portion of the greater Side  $AB$ ; which taken therefrom, the Remainder is  $FB$ : In the middle whereof falleth the Perpendicular  $CG$ ; for  $GB$  and  $GF$  are equal. And thus have we also the Oblique Triangle  $ACB$  divided into the two Right-angled ones  $AGC$ , and  $CGB$ : And so the  $\angle$ 's may be found by *Ax. 1.*

By these three *Axioms*, all the Conclusions of Right-lin'd Triangles may be resolved; and tho' they are chiefly intended for the Oblique-angled, yet are they likewise true in all plain Triangles whatsoever.

This last *Axiom* is of use, when three Sides are given to find an Angle.

\* Vid. the End of the last Note upon Def. 1. §. 2. Chap. 1.

CASE

## CASE I.

The Angles (viz. A and B), and one Side (viz. AC) given, to find the other two Sides.

In the Triangle here used, let the given things be

$$\text{The } \left\{ \begin{array}{l} \angle A \dots 30^\circ 00' \\ \angle B \dots 37^\circ 00' \\ \text{Side AC } 30 \text{ Parts,} \end{array} \right\} \begin{array}{l} \text{the things} \\ \text{requir'd} \end{array} \left\{ \begin{array}{l} \text{Sides.} \\ \text{BC.} \\ \text{and} \\ \text{AB.} \end{array} \right\}$$

1. The Proportion for finding BC (by Ar. 1.) is this:

|                                               |          |
|-----------------------------------------------|----------|
| As 1. $\angle B$ , $37^\circ 00'$ — Ar. Comp. | 0.220537 |
| To the Side AC, 30 Parts,                     | 1.477121 |
| So the s. $\angle A$ , $50^\circ 00'$         | 9.884254 |
| To the Side BC 38.19                          | 1.581912 |

Remember always that (as 'tis said before) in every Right-lin'd  $\Delta$ , the three  $\angle$ 's thereof are equal to two Right  $\angle$ 's, or  $180^\circ$ . So that any two of them being known, the third  $\angle$  is also found, being the Complement or Remainder of the other two to  $180^\circ$ . As here the  $\angle$ 's A and B being known, I say the  $\angle$  C is also known to be  $93^\circ$ : for the Sum of the two  $\angle$ 's A and B, (viz.)  $87^\circ$  being subtracted from  $180^\circ$ , there remains 93, the Quantity of the  $\angle$  C. And therefore having found all the  $\angle$ 's,

2. The Proportion to find the Side AB (by the said Axiom) is

M

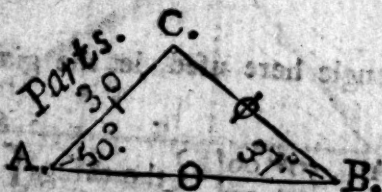
As



As  $s. \angle B, 37^{\circ} 00'$  *Ar. Com.*  
 To the Side  $AC, 30$  Parts,  
 So  $s. \angle C, 87^{\circ} 00'$

0.220537  
 1.477121  
 9.999404  
 -----  
 1.697062  
 -----

To the Side  $AB, 49.78.$



In the last Operation the  $\angle C$  being above  $90^{\circ}$ , I subtract it from  $180^{\circ}$ , and work by the Remainder, viz.  $87^{\circ} 00'$ . The like must be observed in all such Cases.

And here *note*, In finding an  $\angle$  by the Proportion, it will be doubtful sometime whether the  $\angle$  sought be Acute or Obtuse, the Logarithm being the same to both. Nor can this Doubt be resolved better, than by a true Delineation of the Triangle, or else by finding the third  $\angle$ , (as you are before directed) except you had rather work by this

*General Rule.*

See if the Square of the Side subtending the doubtful  $\angle$  exceeds the Sum of the Squares of the other two Sides, then 'tis Obtuse: If they are equal, 'tis Right: But if the Square of the said Side be less than the Sum of the Squares of the other two Sides, the  $\angle$  required is Acute.

*To perform this Case Instrumentally.*

For the first Proportion, which is,

$$\text{As, } s. \angle B : AC :: s. \angle A : BC.$$

$$37^{\circ} 00' : 30p :: 50^{\circ} 00' : 38.19.$$

And

And therefore,

Extend from  $37^{\circ} 00'$ , to  $50^{\circ} 00'$  in the Sines, the same Extent will reach from 30 to 38.19 in the Line of Numbers.

For the second Proportion, which is,

$$\text{As, } s. \angle B : AC :: s. \angle C : AB.$$

$$\frac{37^{\circ} 00'}{30 \text{ p}^{\text{ts}}} :: \frac{87^{\circ} 00'}{49.78}.$$

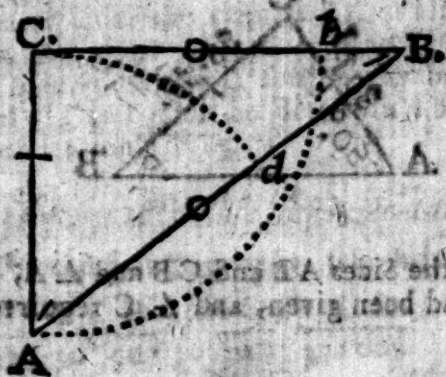
And therefore,

Extend from  $37^{\circ} 00'$  to  $87^{\circ} 00'$  in the Sines, that Extent will reach from 30 to 49.78 in Numbers.

*The Geometrical Performance of this Case is as follows:*

1. Make  $AC = 30$  Parts, and upon A with  $60^{\circ}$  of Chords, describe the Arch  $Cd$ ; upon which set  $50^{\circ} 00'$  from C to  $d$ , and draw the Line  $AB$  thro' the Point  $d$ .

2. With  $60^{\circ}$  of Chords, and one Foot of the Compasses in C, draw the Arch  $Ab$ , and from A to  $b$  set off  $93^{\circ} 00'$ ; \* through which Point  $b$  draw the Line  $CB$ , until it meet with  $AB$  in the Point B, so shall B be the other  $\angle$  of the Triangle. And if  $AB$  and  $CB$  be measured upon your Scale, they will be found to contain 38.19 and 49.78 Parts as before.



M 2

\* Note,

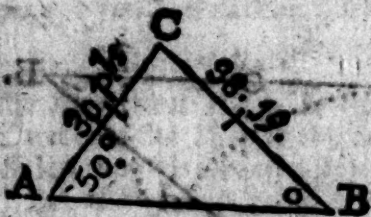
\* Note, Most Lines of Chords running but to  $60^\circ$ , the way to lay down a greater Quantity is by taking the half thereof, and double it upon the Arch; as the half of  $93$  (above) is  $46^\circ 30'$ ; which doubled upon the prick'd Line  $Ab$ , gives the true Quantity or Measure of the  $\angle C$ , viz.  $93^\circ$ .

## CASE II.

Two Sides (viz.  $AC$  and  $CB$ ) with an  $\angle$  (viz.  $A$ ) opposite to one of them being given, to find the  $\angle$  ( $B$ ) opposite to the other Side.

1. The Proportion (by Ar. 1.) to find the  $\angle B$ , is,

|                                                |                 |
|------------------------------------------------|-----------------|
| As the Side $CB$ , $38^\circ 19$ Parts Ar. Co. | 8.418088        |
| To $\angle A$ , $50^\circ 00'$                 | 9.884254        |
| So the Side $AC$ , $30$ Parts,                 | 1.477121        |
| To $\angle B$ , $37^\circ 00'$                 | <u>9.779463</u> |



But if the Sides  $AB$  and  $CB$  and  $\angle A$ , opposite to  $CB$ , had been given, and  $\angle C$  required.

Then

Then the Proportion (by the said Ax.) had been thus;

As, Side CB 38.19 Parts *Ar. Co.* 8.418088

To s.  $\angle A$  50° 00'. 9.884254

So the Side AB, 49.78 Parts. 1.697062

To s.  $\angle C$ , 93° 00'. † 9.999404

† Here now you may more immediately see the absolute Necessity there was for adding the Note upon the last Case foregoing, concerning the Doubtfulness sometimes of the required  $\angle$ : for looking in the Tables of Log. Sines, &c. for the Log. of 9.999404, I find against it 87° 00'; whereas the Angle required is really 93°. But 9.999404 being the Log. Sine to both those  $\angle$ 's, it may be question'd, by a young Beginner, which of them it belongs to. Now in case of any such Doubt, either of the before-mention'd Ways will determine whether it be the Sine of an Acute or Obtuse  $\angle$ . That is, whether the  $\angle$  sought be more or less than 90°; so 9.999404 is discovered to be the Sine of an Obtuse  $\angle$ , the Measure whereof is 93° 00'.

And here observe farther for your Help in this matter, That if the  $\angle$  given be Obtuse, the  $\angle$  requir'd is Acute. And if it be Acute, and oppos'd to the greater Side, the required  $\angle$  is also Acute: But when the given  $\angle$  is Acute, and oppos'd to the lesser Side, then the  $\angle$  sought may be term'd a Doubtful  $\angle$ , and must be determin'd by one of the before-mention'd Ways.

To perform this Case Instrumentally.

For the first Operation, which is,

As, CB : s.  $\angle A$  :: AC : s.  $\angle B$ .

38.19 : 50° 00' :: 30 : 37° 00'.

M 3

And



And therefore

The Extent from 38.19 to 30 in the Line of Numbers, will reach from the Sine of  $50^{\circ} 00'$ , to the Sine of  $37^{\circ} 00'$ .

For the second Operation, which is,

$$\text{As, } CB : s. \angle A :: AB : s. \angle C.$$

$$38.19 : 50^{\circ} 00' :: 49.78 : 93^{\circ} 00'.$$

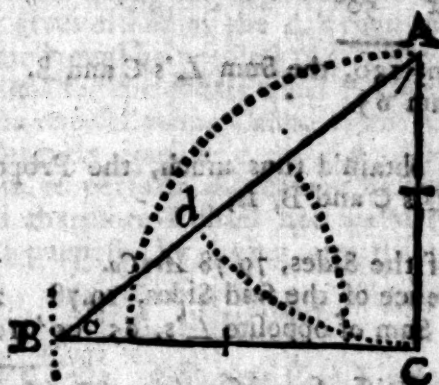
And therefore,

The Extent from 38.19 to 49.78 in Numbers, will reach from the Sine of  $50^{\circ} 00'$  to the Sine of  $87^{\circ}$ , the Supplement whereof to  $180^{\circ}$  is  $93^{\circ}$ , as before.

*The Geometrical Protraction thereof is as followeth.*

Make AC equal to 30 Parts, then with  $60^{\circ}$  of Chords draw the Arch Cd, and from C to d set off  $50^{\circ} 00'$ , and through the Point d draw the Line AB: Again, take 38.19 from your Scales of equal Parts, and setting one Foot in C, with the other cross the Line AB in the Point B, and draw CB; so have you completed the Triangle required. Now to find the Quantity of the  $\angle$  B and C, do thus: Upon B, as a Centre, (with  $60^{\circ}$  of Chords) strike the Arch f, g, which measured upon the Line of Chords, will be found to contain  $37^{\circ} 00'$ . Also if upon C and A you do the same, and measure the several Arches so describ'd, they will give the Quantity of those  $\angle$ 's, viz.  $\angle C$ ,  $93^{\circ}$ :  $\angle A$ ,  $50^{\circ}$ , as before.

CASE



### CASE III.

*Two Sides (viz. AB, and AC) with an  $\angle$  included between them, (viz. A) given, to find the other two  $\angle$ 's.*

The Operation (by Ax. 2.) is as follows.

First, Find the Sum and Difference of the Side AB and AC; also the half Sum of the two opposite  $\angle$ 's; thus,

$$\text{Side } \left\{ \begin{array}{l} \text{AB is } 49.78 \\ \text{AC is } 30.00 \end{array} \right\} \text{Parts}$$

$$\text{Sum } 79.78$$

$$\text{Difference } 19.78$$

Then to find the Sum of the  $\angle$ 's C and B, subtract the  $\angle$  A from  $180^\circ$ , the Remainder is the Sum of the other two (by 1. of Chap. 6.)

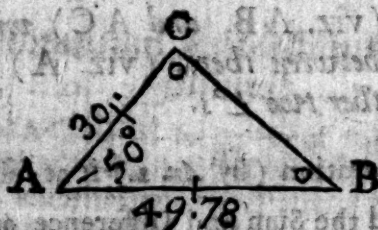
$180^\circ$

Subtract  $180^{\circ}$   
 $50^{\circ}$

Remains  $130^{\circ}$ , the Sum  $\angle$ 's C and B.  
Half Sum  $65^{\circ}$ .

Having obtain'd thus much, the Proportion to find the  $\angle$ 's C and B, is,

|                                                                      |                |           |
|----------------------------------------------------------------------|----------------|-----------|
| As Sum of the Sides, 79.78                                           | <i>Ar. Co.</i> | 7.098106  |
| To Difference of the said Sides, 19.78                               |                | 2.296226  |
| So $\frac{1}{2}$ half Sum of opposite $\angle$ 's, $65^{\circ} 00'$  |                | 10.331327 |
| To $\frac{1}{2}$ the Diff. of opposite $\angle$ 's, $28^{\circ} 00'$ |                | 9.725659  |



Which added to  $\frac{1}{2}$  Sum of the  $\angle$ 's C and B, viz.  $65^{\circ}$ ,  
gives the greater  $\angle$  C  $93^{\circ} 00'$   
And being subtracted, leaves the lesser  $\angle$  B,  $37^{\circ} 00'$

*To perform this Case Instrumentally.*

1. Having found the before-mention'd Requisites as above, and being provided with a Scale, (or rather a Sector) whereon is graduated a Line of Tangents, you may then proceed to solve this Proportion as followeth:

Extend from 79.78 to 19.78 in the Line of Numbers, the same Extent will reach from  $65^{\circ} 00'$  to  $28^{\circ} 00'$ , being number'd backwards, in the Tangents.

gents. Which added to, or subtracted from, &c. as before, gives either of the  $\angle$ 's required.

I presume it must be needless to give any Scheme, or Directions for the geometrical Performance of this and the two following Cases, to any who have but indifferently consider'd the *last*; for he that knows how to perform *that* cannot be ignorant of *these*: And therefore I shall not waste time and room to no purpose, by saying any thing further about it.

### CASE IV.

*Two Sides (viz. AB and CB) with an  $\angle$  included between them (viz. B) given, to find the third Side.*

The Operation by the last foremention'd Axiom, is,

1. First, Find the Sum and Difference of the Sides, and also the half Sum of the two opposite  $\angle$ 's, as before in the last Case; thus,

Side AB, 49.78      186

Side CB, 38.19      37

Sum 87.97       $\frac{1}{2}$  143

Difference 11.39       $71^{\circ} 30' = \frac{1}{2}$  Sum of the  $\angle$ 's A and C.

Then to find the third Side, viz. AC.

Say, As the Sum of the Sides AB and CB 87.97. *Ar Co.* } 7.055665

To their Difference 11.39 } 2.064083

So  $\frac{1}{2}$  of  $\frac{1}{2}$  the Sum of the opposite  $\angle$ 's A and C,  $71^{\circ} 30'$  } 10.475480

Tot. of  $\frac{1}{2}$  the Diff. of those  $\angle$ 's  $21^{\circ} 30'$  } 9.595228

Which



Which added to  $71^{\circ} 30'$  gives the greater  $\angle C$ ,  $93^{\circ} 00'$ ; or being subtracted therefrom, leaveth  $50^{\circ} 00'$  for the lesser  $\angle A$ .

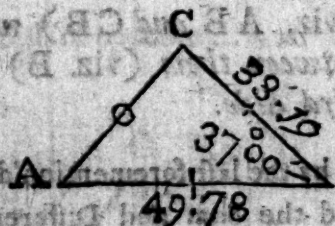
And thus having found all the  $\angle$ 's, you may find the Side AC by the first *Axiom*; saying,

As s.  $\angle A$ ,  $50^{\circ} 00'$  — *Ar. Co.* 0.115746

To the Side CB, 38.19. 1.581912

So s.  $\angle B$ ,  $37^{\circ} 00'$ . 9.779463

To the Side AC requir'd, 30 Parts. 1.477121



The *Instrumental* Performance of the two last Proportions above is thus:

1. Extend from 87.97 to 11.59 in Numbers, the same Extent will reach from  $71^{\circ} 30'$  to  $21^{\circ} 30'$  in the Tangents; which added to, or subtracted from,  $71^{\circ} 30'$  gives the  $\angle$ 's A and C, as before.

2. The Extent from the Sine of  $50^{\circ} 00'$  to the Sine of  $37^{\circ} 00'$ , will reach from 38.19 to 30 in the Line of Numbers.

## CASE V.

The three Sides of any Oblique-angled Triangle given, to find the Angles.

The manner of operating to resolve this Case (by the third *Axiom*) is,

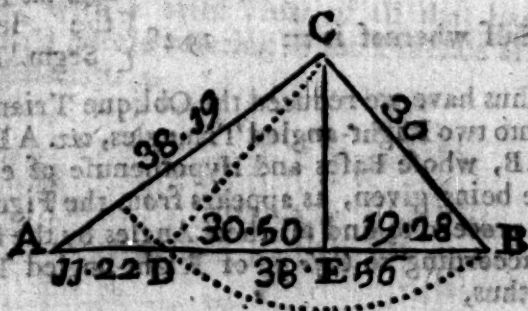
1. To

I. To find the Sum and Difference of the two shortest Sides, viz. AC and CB, which in the Triangle we have all along made use of in resolving the several Cases of Oblique plain Triangles, is as followeth :

$$\begin{array}{l} \bullet \text{ Side AC} = 38.19 \\ \bullet \text{ Side CB} = 30.00 \end{array} \left. \vphantom{\begin{array}{l} \bullet \text{ Side AC} = 38.19 \\ \bullet \text{ Side CB} = 30.00 \end{array}} \right\} \text{Parts.}$$

$$\text{Sum} = 68.19$$

$$\text{Difference} = 8.19$$



Whole Base AB is  $49^{\circ} 78'$ .

Then say,

|                                                                                 |                                                                                                                                                          |          |
|---------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|----------|
| As the greatest Side AB, $49.78$ .                                              | <i>Ar. Co.</i>                                                                                                                                           | 8.302938 |
| To Sum of the other two, $68.19$ .                                              |                                                                                                                                                          | 2.833721 |
| So is the Difference, $8.19$ .                                                  |                                                                                                                                                          | 1.913284 |
| To the Difference of the Segments of the Base (or greater Side) AB; to wit, AD— | $\left. \vphantom{\begin{array}{l} \text{To the Difference of the Segments of the Base (or greater Side) AB; to wit, AD—} \end{array}} \right\} = 11.22$ | 3.049943 |

Which Difference AD, being added to the greater Side AB, gives a Number, half whereof is the greater Segment AE; and being subtracted therefrom leaves a Number, the half of which is the lesser Segment of the Base AB; to wit, EB.

Greater

Greater Side AB, 49.78  
 11.22  
 Difference of the Seg-  
 ments thereof, viz. } — is 61.00  
 AD = 11.22.

Half of which is = 30.50 { the greater  
 Segm. A E.

And being — is 38.56 { the Measure of  
 the other Part of  
 the Base, viz.  
 DB, in the middle  
 whereof falleth  
 the Perpen. CE  
 from the  $\angle$  C.

Half whereof is = 19.28 { the lesser  
 Segm. E B.

And thus have we reduced the Oblique Triangle  
 given, into two Right-angled Triangles, viz. A E C,  
 and C E B, whose Bases and Hypothenuse of each  
 Triangle being given, as appears from the Figure,  
 we may proceed to find all their Angles by the first  
 Axiom, according to Case 5 of Right-angled Tri-  
 angles; thus,

1. In the Triangle A E C.

To find the Angles, the Proportion is,  
 As the Hypothenuse A C, 59.19.  
 To Radius 90°.  
 So is the Base A E, 30.50.  
 To  $\angle$  C, 53° 00'.  
 Whose Complement is the  $\angle$  at A = 37° 00'.

2. In the Triangle C E B.

To find the  $\angle$ 's, say thus:  
 As the Hypothenuse C B 50.  
 To Radius 90°.  
 So is the Base C E, 19.28.  
 To  $\angle$  C, 40° 00'.  
 The Comp. whereof to 90° 00' is the  $\angle$  B = 50°.  
 And

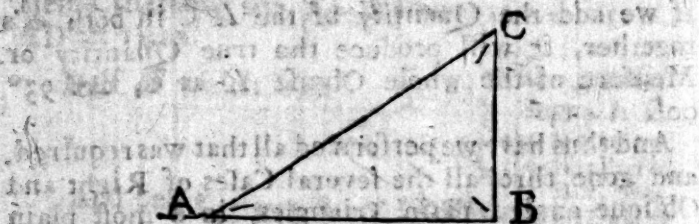
And now having found the  $\angle$ 's A and C in the first Triangle to be  $37^\circ$  and  $53^\circ$ , and the  $\angle$ 's B and C in the second Triangle to be  $50^\circ$  and  $40^\circ$ , if we add the Quantity of the  $\angle$  C in both  $\Delta$ 's together, it will produce the true Quantity or Measure of the whole Obruse  $\angle$  at C, viz.  $93^\circ 00'$ .

And thus have we performed all that was required, and gone thro' all the several Cases of Right and Oblique-angled plain Triangles, in a most plain and familiar manner.

But before I end this Chapter, I shall present the Learner with a brief View of all that has been mention'd herein relating to the Practice of plain Trigonometry.

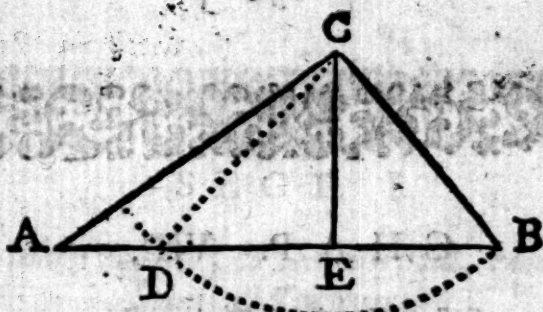


*A Summary, or brief View of the Doctrine of plain Triangles.*



Right-angled.

| <i>Cases.</i> | <i>Things given.</i>                                      | <i>Things required.</i>                        | <i>Proportions.</i>                                                                                                                                                                     |
|---------------|-----------------------------------------------------------|------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1             | $\angle$ 's $\begin{cases} A \\ C \end{cases}$<br>S. A C. | Base<br>A B.                                   | $1. \text{As Rad} : AC :: s. \angle C :$<br>$2. \text{As } se. \angle A : AC :: \text{Rad} :$<br>$3. \text{As } se. \angle C : AC :: t. \angle C :$                                     |
| 2             | $\angle$ 's $\begin{cases} A \\ C \end{cases}$<br>S. A B. | Perpen<br>C B.                                 | $1. \text{As } s. \angle C : AB :: s. \angle A :$<br>$2. \text{As Rad} : AB :: t. \angle A :$<br>$3. \text{As } t. \angle C : \text{Rad} :: AB :$                                       |
| 3             | $\angle$ 's $\begin{cases} A \\ C \end{cases}$<br>S. A B. | Hypoth<br>A C.                                 | $1. \text{As } s. \angle C : AB :: \text{Rad} :$<br>$2. \text{As Rad} : AB :: se. \angle A :$<br>$3. \text{As } t. \angle C : AB :: se. \angle C :$                                     |
| 4             | S. A B.<br>and<br>S. B C.                                 | $\angle$ 's $\begin{cases} A \\ C \end{cases}$ | $1. \text{As } AB : \text{Rad} :: BC : t. \angle A.$<br>Whose Complem. is the $\angle C.$<br>$2. \text{As } BC : \text{Rad} :: AB : t. \angle C.$<br>Whose Complem. is the $\angle A.$  |
| 5             | S. A C.<br>and<br>S. A B.                                 | $\angle$ 's $\begin{cases} C \\ A \end{cases}$ | $1. \text{As } AC : \text{Rad} :: AB : s. \angle C.$<br>Whose Complem. is the $\angle A.$<br>$2. \text{As } AB : \text{Rad} :: AC : se. \angle A.$<br>Whose Complem. is the $\angle C.$ |
| 6             | S. A B.<br>and<br>S. B C.                                 | Hypoth<br>A C.                                 | $1. \text{As } BC : \text{Rad} :: AB : t. \angle C.$<br>Again :<br>$2. \text{As } s. \angle C : AB :: \text{Rad} : AC.$                                                                 |
| 7             | S. A B.<br>and<br>S. A C.                                 | Perpen<br>C B.                                 | $1. \text{As } AC : \text{Rad} :: AB : s. \angle C.$<br>Whose Compl. is the $\angle A.$<br>$2. \text{As Rad} : AC :: s. \angle A : CB.$                                                 |



Oblique Angled.

| Cases | Things given.                                                                                                                | Things required.                                             | Proportions.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|-------|------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1     | $\angle$ 's $\begin{cases} A & S. B C \\ B & \text{and} \\ S. A C & S. A B. \end{cases}$                                     |                                                              | 1. As $s. \angle B : AC :: s. \angle A : BC$ .<br>The $\angle$ 's A and B being given, the $\angle$ C is known, & therefore<br>2. As $s. \angle B : AC :: s. \angle C : AB$ .                                                                                                                                                                                                                                                                                                                      |
| 2     | $\begin{cases} S. A C. \\ \text{and} \\ S. C B. \end{cases}$                                                                 | $\angle B.$                                                  | As $CB : s. \angle A :: AC : s. \angle B$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 3     | $\begin{cases} S. A B. \\ S. A C. \\ \text{and} \\ \angle A. \end{cases}$                                                    | $\angle$ 's $\begin{cases} C \\ \text{and} \\ B \end{cases}$ | As Sum $AB \& AC : \text{Diff. } AB \& AC :: r. \frac{1}{2} \text{ Sum } \angle$ 's C & B : $r. \frac{1}{2} \text{ Difference } \angle$ 's C and B.<br>Which $\begin{cases} + \\ - \end{cases} \frac{1}{2} \text{ Sum } \begin{cases} \text{greater } \angle C. \\ \text{lesser } \angle B. \end{cases}$                                                                                                                                                                                           |
| 4     | $\begin{cases} S. \begin{cases} AB \\ CB \end{cases} \\ \text{and} \\ \angle B. \end{cases}$                                 | $S. A C.$                                                    | Having found (by the last Case) $\angle$ 's C & B, and consequently $\angle A$ ; then (by 1 <sup>st</sup> Axiom) say,<br>As $s. \angle C : AB :: s. \angle B : AC$ .                                                                                                                                                                                                                                                                                                                               |
| 5     | $\begin{cases} S. \begin{cases} AB \\ AC \\ CB \end{cases} \\ \angle$ 's $\begin{cases} A \\ C \\ B \end{cases} \end{cases}$ | $\begin{cases} A \\ C \\ B \end{cases}$                      | As the greatest Side $AB$ ,<br>To Sum of $AC$ and $CB$ ;<br>So their Diff. to Diff. of the Segments of the Base, viz. $AD$ .<br>$\begin{matrix} + 2 AB \\ - 2 AC \end{matrix} \frac{1}{2} \begin{cases} AE \text{ great } \angle \\ EB \text{ lesser } \angle \end{cases} \text{ Seg.}$<br>And thus have we the Oblique $\triangle$ resolved into two Right $\triangle$ 's; in either of which the Hypotenuse and Base is known; and thence $\angle$ 's may be found by the 1 <sup>st</sup> Axiom. |



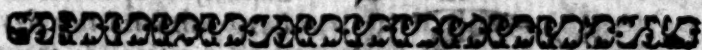
B A  
C H A P. VIII.

Of Spherical Triangles.

AS before in Plain, so here in *Spherical Triangles*, I shall first of all lay down, and demonstrate (when necessary) such general Definitions, Rules, Theorems, and Properties of Spherical Triangles, as will enable the young Learner the better to apprehend and conceive aright of the whole Doctrine of the Dimensions of those kind of Triangles; and also shall explain such Affections of the Circles of the Sphere, and Problems deduced from thence, as are requisite for the geometrical Performance hereof. And next shall proceed to acquaint him with the Solution of all the Cases in their Order; and that three several ways as before, viz. *Arithmetically, Instrumentally, and Geometrically*. In the doing whereof, I shall make use of no other *Axiom* to be admitted as the Ground-work of all, but that of the Lord *Nepier* (Baron of Merchiston in Scotland) which he calls his *Catholick, or Universal Proposition*: By which alone all the Cases of *Spherical Trigonometry* may be resolved with very little Trouble, or Burthen to the Memory, and which therefore I recommend to the Learner as sufficient for his purpose.

The Explanation and Use whereof follows next before the Operation of the several Cases, &c.

S E C T.



## S E C T. I.

*Of things necessary to be understood, relating to Spherical Trigonometry.*

## D E F I N I T I O N I.

**A** Spherical Triangle is comprehended and formed by the Conjunction and Intersection of three great Circles of the Sphere, described in the Superficies thereof. So that the three Sides of a Spherical  $\Delta$  are three Parts or Arches of so many great Circles mutually intersecting each other; and are either of them less than a Semicircle, or 180 Degrees, being measured by a Scale or Arch of equal Degrees.

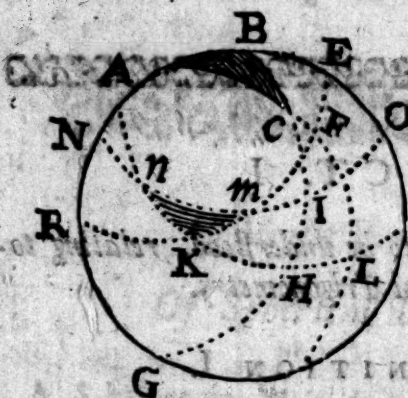
Therefore the Arches of Parallels, or other lesser Circles of the Sphere, are not to be taken as the Sides of a Spherical  $\Delta$ .

**DEF. II.** The Measure of a Spherical  $\angle$ , is an Arch of a great Circle described on the angular Point, and intercepted between the two Sides, being produced (if need be) until they be Quadrants, viz. 90 Degrees each.

Thus in the Spherical  $\Delta ABC$ , the Measure of the  $\angle A$  is not the Arch  $BC$ , but the Arch  $EF$  intercepted between the two Sides  $AB$  and  $AC$ , continued till they are Quadrants, viz. to  $E$  and  $F$ ; because the Arch  $BC$  is not described upon  $A$ , as a Centre, but the Arch  $EF$  is:

Therefore the Arch  $BC$  cannot be the Measure of the  $\angle A$ , (by Def. 13. c. 1.)





DEF. III. The Sides of a Spherical  $\Delta$  being continued till they meet together, do make two Semicircles, and at their Intersection do comprehend an  $\angle =$  and opposite to the first  $\angle$ .

Thus in the  $\Delta ABC$ , the Sides BA and BC being continued to G, do make the Semicircles BAG & BCG, comprehending the  $\angle AGC = \angle ABC$ ; because the Arch NI (being distant from G and B  $90^\circ$ ) is the Measure of both those  $\angle$ 's, (by the last.) And not only so, but the Sides and  $\angle$ 's of one  $\Delta$  are the Complements of the Sides and  $\angle$ 's of the other; as the Side CG is the Complement of BC to a Semicircle, and the  $\angle ACG$  is the Complement of  $\angle ACB$  to a Semicircle, or  $180^\circ$ . Understand the like of the rest. (See Def. 2. c. 6. of plain  $\Delta$ 's.)

DEF. IV. Whence it may be gathered, That every Spherical  $\Delta$  hath, opposite to each angular Point, another having the same Base with the former, and the  $\angle$  opposed thereto equal; and that the other Parts of it are Complements of the several Parts of the former to a Semicircle. Which needs no further Proof.

DEF. V The Sides of a Spherical  $\Delta$  may be changed into  $\angle$ 's, and the contrary; the Complement of the greatest Side, or greatest  $\angle$  to a Semicircle, being taken for the greatest Side, or greatest  $\angle$ .

As in the  $\Delta ABC$ , Obtruse  $\angle^d$  at B, let EF be the Measure of the  $\angle$  at A; OI, the Measure of the

the Acute  $\angle$  at B, (being the Complement of the Obtuse  $\angle$  at B, the greatest  $\angle$  in the  $\triangle ABC$ ); and let HL be the Measure of the  $\angle$  at C. Now,

1.  $Km = FE$ , because KF and  $mE$  are Quadrants, and their common Complement is  $mF$ .

2.  $n m = IO$ , because  $mO$  and  $In$  are Quadrants, and their common Complement is  $mI$ .

3.  $AK = KL$ , because KL and  $nH$  are, &c. and their common Complement is KH. Therefore the Sides of the  $\triangle Kmn$ , =  $\angle$ 's of the  $\triangle ABC$ ; taking for the greatest  $\angle ABC$  the Complement thereof, viz.  $IBO$ .

And by the Converse hereof, it may also be demonstrated, that the Sides of the  $\triangle ABC = \angle$ 's of the  $\triangle Kmn$ . For the

$$\text{Side } \begin{cases} AB \\ BC \\ AC \end{cases} = \begin{cases} NR \\ HI \\ FL \end{cases} \text{ the Measure of the } \angle \begin{cases} n m K. \\ K n m. \\ FKL. \end{cases}$$

Which is the Complement of the  $\angle nKm$ .

For that  $\begin{cases} AF \text{ and } CL \\ AR \text{ and } NB \\ BI \text{ and } HC \end{cases}$  are Quadrants,  $\begin{cases} CF. \\ AN. \\ CI. \end{cases}$  and their common Compl. is  $\begin{cases} CF. \\ AN. \\ CI. \end{cases}$

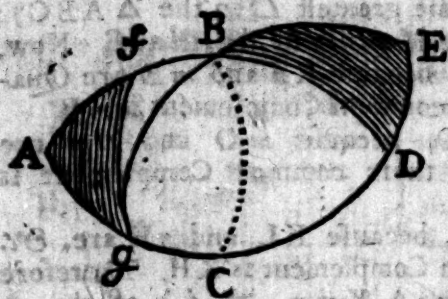
Therefore the Sides may be turned into  $\angle$ 's; and the contrary: Which was to be proved.

DEF. VI. If a Spherical  $\triangle$  have one or more right  $\angle$ 's, it is called a Right-angled Spherical  $\triangle$ .

DEF. VII. If a Spherical  $\triangle$  have one or more of its Sides Quadrants, it is called a Quadrantal  $\triangle$ .

DEF. VIII. If it have neither Right  $\angle$ , nor any Side a Quadrant, it is called an Oblique Spherical  $\triangle$ .

DEF. IX. If a Spherical  $\triangle$  be both Right-angled and Quadrantal, the Sides thereof are equal to the opposite  $\angle$ 's. As



As in the  $\Delta ABC$ , whose 3  $\angle$ 's A, B and C are all right  $\angle$ 's, and the Sides subtending those  $\angle$ 's Quadrants; for A is the Pole of the great Circle BC, B the Pole of AC, and C the Pole of AB; therefore the Sides are Quadrants, and also equal to the opposite  $\angle$ 's.

DEF. X. The Sides of a Right-angled Spherical  $\Delta$ , with two Acute  $\angle$ 's, are every one of them less than a Quadrant; as in the  $\Delta fAg$ , whose Sides are plainly less than Quadrants.

DEF. XI. Hence it must needs appear evident, that the two Sides of a Right  $\angle^d$  Spherical  $\Delta$ , with two Obtuse  $\angle$ 's, are greater than Quadrants, and the third Side is less than a Quadrant; as in the  $\Delta fDg$ , Obtuse  $\angle^d$  at f and g, the Sides fD and gD, opposite to the two Obtuse  $\angle$ 's f and g, are each of them greater than Quadrants; but the Side fg subtending the Right  $\angle$  at D, is lesser.

DEF. XII. A Right  $\angle^d$  Spherical  $\Delta$ , with two Acute  $\angle$ 's, hath from the Right  $\angle$ , a Right  $\angle^d$   $\Delta$  opposite thereunto, with two Obtuse  $\angle$ 's; as the  $\Delta$ 's fAg, and fDg do plainly make appear.

DEF. XIII. If the third  $\angle$  of a Spherical  $\Delta$ , having two Right  $\angle$ 's, be Acute, the third Side is less than a Quadrant; but if Obtuse, then it's greater than a Quadrant.

DEF.

DEF. XIV. If any  $\angle$  of a  $\Delta$  be nearer to a Quadrant than its opposite Side, two Sides of that  $\Delta$  will be of one kind, and the third less than a Quadrant; & contra: as the  $\Delta f D g$ , whose  $\angle$  at  $D$  is nearer to a Quadrant than its opposite Side  $fg$ , hath therefore both the Sides including the  $\angle D$ , viz.  $fD$  and  $gD$ , alike; that is, both of one kind.

DEF. XV. An Oblique  $\Delta$  consisteth simply of Acute or Obtuse  $\angle$ 's, or of both.

DEF. XVI. A Spherical  $\Delta$ , with two Obtuse  $\angle$ 's and one Acute  $\angle$ , is opposite to a Spherical  $\Delta$  simply Acute  $\angle$ 's; & contra.

As if the  $\angle$ 's at  $A$  and  $D$  be supposed Acute, then the  $\Delta f D g$ , with two Obtuse  $\angle$ 's at  $f$  and  $g$ , and one Acute  $\angle$  at  $D$ , is opposite to the simple Acute  $\angle$ 's  $\Delta A f g$ .

DEF. XVII. A Spherical  $\Delta$ , with two Acute  $\angle$ 's and one Obtuse, is opposite to a Spherical  $\Delta$  simply Obtuse  $\angle$ 's; & contra.

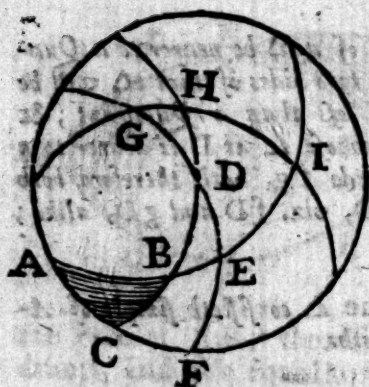
As, suppose the  $\angle$ 's  $A$  and  $D$  to be Obtuse; then the  $\Delta f A g$ , with the two Acute  $\angle$ 's  $f$  and  $g$ , and one Obtuse  $\angle$  at  $A$ , is opposite to the simply Obtuse  $\angle$ 's  $\Delta f D g$ .

DEF. XVIII. The three  $\angle$ 's of every Spherical  $\Delta$  are greater than two Right  $\angle$ 's.

This is evident in Spherical  $\Delta$ 's having more Right or Obtuse  $\angle$ 's than one: But in Acute  $\angle$ 's  $\Delta$ 's it may be thus demonstrated.

In



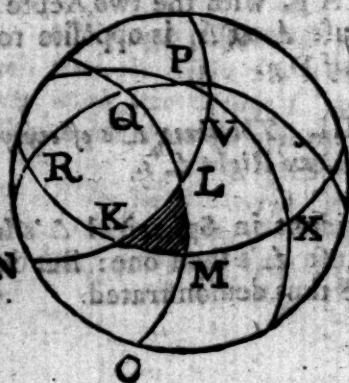


In the Right  $\angle^a$   
Spherical  $\triangle$ , ABC  
Right  $\angle^a$  at C, and  
Acute  $\angle^a$  at A and  
B.

The Measure of the Acute  $\angle$   $\left\{ \begin{array}{l} BAC \\ ABC \\ or \\ DBE \end{array} \right\}$  is the Arch.  $\left\{ \begin{array}{l} EF \\ HI \end{array} \right\}$  by the 2d hereof.

But the Arches EF and ED together are equal to a Quadrant; therefore the Arches EF and HI added together, are more than a Quadrant; and consequently the  $\angle$ 's answering to those Arches, *namely*, the  $\angle$ 's BAC and ABC together, are more than a Quadrant. But the  $\angle$  ACB is a Right  $\angle$ ; therefore the  $\angle$ 's together are greater than two Right  $\angle$ 's.

Again, in the Acute  $\angle^a$   $\triangle$  KLM.



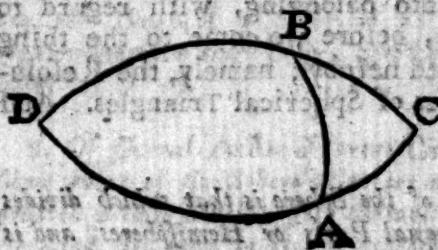
The

The Measure of  $\begin{Bmatrix} KLM \\ MKL \\ LMK \end{Bmatrix}$  is the Arch  $\begin{Bmatrix} NO \\ VX \\ RQ \end{Bmatrix}$   
 the Acute  $\angle$

But these three Arches NO, VX, RQ together, are more than two Quadrants. For PQ and PV (being the Complements of the two Arches QR and VX) added together, are less than the Arch NO, the Measure of the third  $\angle$ ; consequently the three Arches, which are the Measures of the three  $\angle$ 's, are more than two Quadrants. Q.E.D.

DEF. XIX. If two Sides of a Spherical  $\Delta$  be equal to a Semicircle, the two  $\angle$ 's at the Base shall be  $\equiv$  two Right  $\angle$ 's; but if they be less, then the two  $\angle$ 's shall be less; but if greater, then shall the two  $\angle$ 's be greater than two Right  $\angle$ 's. This is so easy to conceive, that there needs no further Proof.

DEF. XX. The Sum of the three Sides of a Spherical  $\Delta$ , is less than a whole Circle, or  $360^\circ$ .



For BA is less than BC and AC; therefore DB, DA and BA together, are less than DBC and DAC.

SECT.

## S E C T. II.

*Of such Affections and Properties of great Circles of the Sphere, as relate to the Solution of Spherical Triangles in general, and are absolutely necessary to be understood.*

**E**VERY Spherical Triangle being (as you have heard) comprehended and form'd by the Intersection of the Parts, or Arches, of three great Circles of the Sphere, or Globe, described on the Surface thereof; it must of course follow, that we should next proceed to acquaint the Learner what a great Circle of the Sphere is, and of the chief Properties thereunto belonging, with regard to our present Design, before we come to the thing principally intended hereby; namely, the Resolution of all the Cases of Spherical Triangles. And first,

1. A great Circle of the Sphere is that which divides the Sphere into two equal Parts or Hemispheres, and is every where distant from its Poles by a Quadrant, or fourth Part of a great Circle, =  $90^{\circ}$ . And such are the Equator, the Meridian, the Horizon, the Ecliptick, &c. the several Positions and Uses of which, in Astronomy, Geography, Navigation, Dialling, &c. are best learnt from the Globe itself; and therefore I forbear to make any further mention thereof, with relation to any of those Sciences, as not being suitable to my Purpose in this place.

2. If a great Circle pass through the Poles of another great Circle, it cutteth it at right  $\angle$ 's; and the contrary.

Note, The Pole of any great Circle of the Sphere is always  $90^\circ$  distant in every Part thereof.

3. All great Circles upon the Globe cut each other into two equal Parts; for their common Section is a Diameter of the Sphere, and consequently the two Sections of the Peripheries of two great Circles are at the Distance of a Semicircle, or  $180^\circ$ . Hence it is, that

4. Every Side of a Spherical  $\Delta$  is less than a Semicircle, as was observ'd before in Def. 1.  $\S$ . 1.

5. If any two great Circles of the Sphere intersect, or cut each other, the opposite  $\angle$ 's at their Intersections are equal. See this proved, Def. 3. of Sect. the last.

6. An Arch of a great Circle is the shortest Distance between two Points on the Superficies of a Globe; and so any two Sides of a Spherical  $\Delta$  taken together, are greater than the third Side. See Def. 3. c. 6. of plain  $\Delta$ 's.

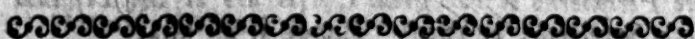
7. Of several Arches of great Circles falling from the same Point of the Sphere's Surface on another Circle, the greatest is that which passeth through the Pole of the Circle, and the next to this is greater than that which is farther off.

8. When one Circle falls upon another, or when the Arches of two great Circles of the Sphere intersect, or cut each other, the Sum of the  $\angle$ 's made thereby is  $=$  to two Right  $\angle$ 's.

Thus having, in the two foregoing Sections, set down and explain'd (when needful) the most principal



cipal Definitions and Affections of Spherical  $\Delta$ 's, and also such Properties of great Circles of the Sphere, as relate to the Solution thereof: I shall in the next, according to my Promise, give you a few easy Problems deduced from thence, by which you will be able to apprehend and perform all the Cases of Spherical  $\Delta$ 's geometrically, without which the Knowledge thereof would be very difficult.



### S E C T. III.

*Containing several Geometrical Problems, of great Use to be learnt, in resolving all the Cases of Spherical Triangles by Projection.*

**B**EFORE I come to give you the Problems here mention'd, and to lay down the Rules for their Operation, it will be a great Help to you, when you come to perform the Work, to understand and have in remembrance the following Observations, which I have thought fit to begin with for this Reason. The Observations are these:

1. You are to take notice, that in the manner hereby intended, for resolving of Spherical  $\Delta$ 's, there are four Sorts of Circles of the Sphere principally to be regarded, viz. the *Primitive Circle*, a *Right Circle*, an *Oblique Circle*, and a *Parallel Circle*.

2. The *Primitive Circle* is the outermost Circle of all, and is that which contains all the rest within it; and in the way we here intend to use it, may be call'd the *Meridian of the Place*: It is represented in the \* Scheme by the Letters Z  $\odot$  N H.

---

\* See Fig. 3. on the Sheet to fold out.

3. A Right Circle is a strait Line or Diameter of the Primitive Circle, which cuts the Primitive in two opposite Points, and divides it into two equal Parts, as  $H \odot$  the Horizon, or  $\overline{AEQ}$  the Equator, &c.

4. An Oblique Circle cuts the Primitive at two opposite Points, but divides it not into two equal Parts; as  $Z \text{ } r \text{ } N$ , an Azimuth; or  $PCS$ , a Meridian.

5. A Parallel Circle is a lesser Circle, neither cutting the Primitive in opposite Points, nor dividing it into two equal Parts, as  $e m f$ , a Parallel of Declination.

All which Circles have their respective Centres and Poles; which must be first found before the great Circle, which constitutes the  $\Delta$  that concerns your Question, can be described, or the Sides or  $\angle$ 's of it measured: And therefore I shall shew,

# PROBLEM I.

*How to find the Centre of any Circle.*

1. The Primitive Circle is always drawn with the Chord of  $60^\circ$ . upon the Centre  $A$ .

2. A Right Circle (being a strait Line) hath no Centre.

3. The Centre of an Oblique Circle always falls in that Right Circle, which cuts the said Oblique Circle in two equal Parts. Thus the Centre of the Circle  $Z \text{ } r \text{ } N$  falls on the Right Circle  $H A \odot$ , viz. at  $\odot$ ; and the Centre of the Circle  $PCS$  falls on the Right Circle  $\overline{AEAQ}$ , viz. at  $\times$ ; which Centres, and also all other Centres of great Circles of the Sphere, are thus found:

Set the Tangents of their Quantities from the Centre of the Primitive Circle, or the Secants of their Quantities from the Circles themselves, upon that straight Line which cutteth the great circular Arch at Right  $\angle$ 's. This Rule is general, when you have given the Quantity of the  $\angle$  which your great Circle is to make with the primitive Circle.

*Example.* Let it be required to describe the Circle Z N making an  $\angle$  at the primitive Circle  $= 47^{\circ} 29'$  min.

Set the Tangent of  $47^{\circ} 29'$  from A upon the Line HA  $\odot$ , the same will reach to the Point  $\odot$ , the Centre of the Circle required. Or the Secant of  $47^{\circ} 29'$  being taken between your Compasses, and one Foot plac'd in Z or N, the other being turned about till it cross the Line HA  $\odot$ , will give you the Point  $\odot$ , the Centre of the said Circle as before.

But if you have three Points through which the Circle's to pass, the Centre is found by *Problem 18. of Chap. 2.*

4. For the Centre of a parallel Circle, having its Distance from the Right Circle to which it is a Parallel, given; set off the Chord of that Distance upon the Primitive Circle from each End of the Right Circle, and the half Tangent of the said Distance from the Centre A upon that Right Circle, which cuts the other at Right  $\angle$ 's. So have you three Points through which the Parallel is to pass to find its Centre, by *Prob. above.*

Or thus: Suppose I would find the Centre of the Parallel of Declination *emf*, which is  $22^{\circ} 30'$ . take the Tangent of the Complement thereof, viz.  $67^{\circ} 30'$ , and set from *e* or *f* to *x*, upon the Line SP continued; or the Secant of the same from A upon the said Line: Either way will give the Centre required.

P R O B-

## PROBLEM II.

*To find the Pole of any Circle.*

By the Note upon Def. 2. of Sect. the last, the Pole of any great Circle is always  $90^\circ$  distant from its Periphery: Therefore,

1. The Pole of the Primitive Circle is always the Centre.

2. The Pole of a Right Circle is always on the Primitive Circle, and may be thus found.

*The Pole of the Circle Z A N is required.*

RULE. From the Chords lay down  $90^\circ$  on the Primitive Circle from Z or N, both ways to H or O; so is H or O the Pole sought, &c.

3. For the Pole of an Oblique Circle, as suppose Z r N.

RULE. The Pole of an Oblique Circle always falls within the Primitive Circle on a Diameter or Right Line, which passeth through its Centre, and cuts the Oblique Circle at Right  $\angle$ 's. Therefore, H, O being first drawn at Right  $\angle$ 's to the Oblique Circle Z r n; if you lay a Ruler from either End of the Arch, viz. Z or N, to r, the Point of Intersection, with the Right Line H O, the same shall give you the Points a and i on the Primitive Circle; which is call'd reducing of r to the Primitive.

Then take  $90^\circ$  of the Chords, and set from a or i, to b and i; a Ruler laid from b to N, or from i to Z, either way, will cut the Right Line H O in the Point k, the Pole of the Oblique Circle Z r N. And after the same manner may be found the Point k, the Pole of the Circle P C S.



Or it may be found thus: Set the half Tangent of the Complement of  $A\gamma$ , or  $Al$ , from  $A$ , on their respective Diameters, it will give you the Poles  $n$  and  $k$ , as before.

4. The Pole of a parallel Circle is always in the Pole of that Right Circle to which it is a Parallel: Thus, the Pole of the Parallel  $emf$  is at  $P$ , the Pole of the Right Circle  $\overline{EAQ}$ .

Note here as before; An Arch of a parallel Circle is never made a Side of a Spherical  $\Delta$ , they being all composed of Arches of great Circles.

Thus have I shewed how to find the Centres and Poles of the several Circles of the Sphere, that are applicable to the Doctrine of Spherical Trigonometry: The next Work will be to shew you, how to describe any kind of Spherical  $\Delta$ , and to measure its Parts, which I shall do in the following Problems.

### PROBLEM III.

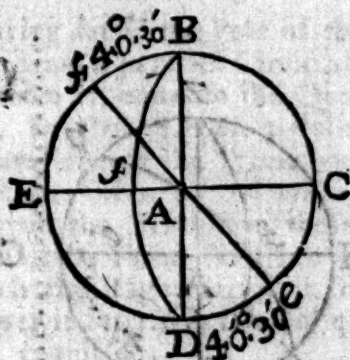
*To describe a Spherical Triangle, or to project any given Angle.*

In this Problem are five Varieties or Cases.

**Case 1.** To make an Angle of any Number of Degrees, (suppose  $40^\circ 30'$ ) whose angular Point may be at the Centre of the Primitive Circle.

**RULE.** With  $60^\circ$  of the Line of Chords, draw the Primitive Circle  $BCDE$ , crossing it with the two Diameters  $BD$  and  $EC$ ; then make  $Dc = 40^\circ 30'$ , and draw the Line  $fe$ , and it's done; for  $Bf$ , and  $De$ , do either of them constitute, at the Centre  $A$ , an  $\angle$  of  $40^\circ 30'$ : Which was required.

*Case*



*Case 2.* To project an Angle of any Number of Degrees and Minutes, (suppose  $46^{\circ}.30'$ .) whose angular Point may be at the Primitive Circle.

**RULE.** Set the Tangent of  $46^{\circ}.30'$ . the given  $\angle$ , from A to C; then upon C, as a Centre, with the Extent CB, or CD, draw the Oblique Circle BfD, it will make at B or D an  $\angle = 46^{\circ}.30'$ .

*Case 3.* To project any given  $\angle$  within the Primitive, but not at the Centre.

Here let it be carefully observ'd as a *general Rule*, That the Centres of all *Oblique Circles* that pass through the same Point in the *Right Circle*, which cuts them at Right  $\angle$ 's, do fall in a *Right Line* drawn through the Centre of the said *Oblique Circle*, and at Right  $\angle$ 's with the said *Right Circle*.

For Example: It is required to draw the Oblique Circle yfp, through the Point f, in the Right Circle HAC, to make with it an  $\angle = 55^{\circ}$ .

Lay a Ruler from Z to f, it will cut the Primitive in y; the Arch Hy measured upon the Chords is  $= 48^{\circ}$ . (the Measure of the  $\angle HZf$ ) therefore place the Tangent of  $48^{\circ}$ . from A to C, and thro' C at

**Case 4.** To draw a Right Circle to make with an Oblique Circle any given  $\angle$ .

**RULE.** This is indeed but the Reverse of the last *Case*, as appears very plain, if what follows be duly consider'd. That the Distance between the Poles of any two great Circles (measured upon an Arch of a great Circle, drawn between the said Poles) is equal to the  $\angle$  made by the Intersection of the said Circles, and consequently to project the said given  $\angle$ , is the direct contrary to the former

Case; for having found the Pole of the given Oblique Circle, draw a parallel Circle about the said Pole distant from it, equal to the Quantity of the  $\angle$  proposed, by *Prob. 8. Case 3.* following; and where that Parallel cuts the Primitive shall be the Pole of the Right Circle; from which set off  $90^\circ$ . both ways upon the Primitive, these Points and the Centre shall be three Points in a strait Line, through which the Right Circle is to be drawn; and where-ever it intersects the Oblique Circle, it makes the  $\angle$  requir'd.

I have not added any Scheme here, because I think it needless.

There is yet another Variety belonging to this Problem; and that is,

*Case 5.* To draw an Oblique Circle to make a given  $\angle$  at the Primitive with another Oblique Circle; as suppose I would draw an Oblique Circle to make at P an  $\angle$  of  $48^\circ$ , with the Oblique Circle SnP. See *Fig. 5th on the Leaf folded out at the End.*

Reduce *n* (the Point where the Oblique Circle cuts the Right Circle in which its Centre falls) to the Primitive, by laying a Scale from P to *n*, which will cut the Primitive in *r*; set the given  $\angle$   $48^\circ$ . of the Chords from *r* to *x*, a Ruler laid from P to *x* will cut the aforesaid Right Circle in *I*; draw the Circle PIS, by *Prob. 1. Case 3.* it is the Oblique Circle required.



## PROBLEM IV.

*To draw a great Circle perpendicular to, or at, Right  $\angle$ 's with a given great Circle.*

*A General Rule.*

Draw a great Circle through the Pole of another great Circle, and it will be at Right  $\angle$ 's with it. See *Defin. 2. §. 2.*

In this *Problem* are four Varieties or Cases.

*Case 1.* To draw a Right Circle at Right  $\angle$ 's with the Primitive Circle HZON, is no other than to draw a Diameter as HO, or ZN, &c. for the Centre of the Primitive is their Pole.

*Case 2.* To draw a right Circle perpendicular to a right Circle. This is no more but to cross the Diameter at Right  $\angle$ 's, or quartering the Primitive Circle with two Diameters.

Thus, if you lay  $90^\circ$ . of the Chords on the Primitive from Z, or from N, to H and O; and draw HAO, it is done.

*Case 3.* To draw an Oblique Circle, as SP, at Right  $\angle$ 's to a Right Circle given, fAq.

**RULE.** Draw the Oblique Circle through the Poles P and S; which you may do from the Centre X, or any other Point on the Line fq, and the Distance P or S; for any Circle so drawn, will be perpendicular to the Right Circle fAq.

But if it be required that the Oblique Circle shall make an  $\angle =$  to any Number of Degrees and Minutes with the Primitive, suppose  $21^\circ$ .

Then

Then draw it by *Case 2. of Prob. 3.* Or thus: With the Secant of  $21^\circ$ , and one Foot in P or S, cross the Line  $f q$  in  $x$ ; so will  $x$  be the Centre of the Circle  $S n P$ , required to be drawn at Right  $\angle$ 's with  $f q$ , and to make an  $\angle$  with the Primitive at  $P = 21^\circ$ .

*Case 4.* To draw an Oblique Circle at Right  $\angle$ 's with an Oblique Circle, as  $S/P$ , in the last Fig.

**RULE.** Find  $b$ , the Pole of the Oblique Circle  $S/P$ , and through  $b$  draw the Circle  $N b Z$ , cutting the Primitive into two equal Parts, and it's done. See *Def. 2. §. 2.* hereof.

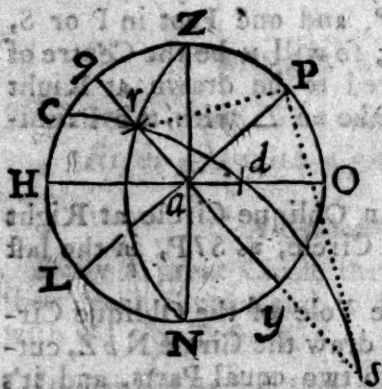
*Note.* If the Point in the given Oblique Circle be limited, then work thus:

Draw a Line through the Centre of the Primitive Circle, and also through the remotest of the two given Points; produce that Line 'till it cuts the Primitive: cross it at Right  $\angle$ 's with another Line through the Centre; and where this last Line cuts the Primitive, draw a Line from that Point, and through the given Point which the other Line passed through. On this last Line, from the Point in the Primitive, erect a Perpendicular to cut the first Line without the Primitive; which crossing is a third Point: and how to describe a Circle which shall pass through any three Points (not in a strait Line), you are directed by *Prob. 18. Chap. 2.* To explain this matter farther, take the following Instance.

*A general Rule  
for drawing an Arch  
of a great Circle  
thro' any two Points  
given within the  
Primitive Circle.*

Let it be required to draw an Arch of a great Circle at Right  $\angle$ 's, with the Oblique Circle  $Z n N$ , to pass through  $d$ , the Pole of the said Oblique Circle and  $r$ , another Point upon the Periphery.

Through



Through A the Centre of the Primitive, and *r* the farthest of the two Points, draw the Line *qry*, extended at pleasure; and draw *PL* at Right  $\angle$ 's therewith. Draw also *rP*, and from *P* erect the Perpendicular *Pp*, cutting the Line *qy* (extended) in *S*; then through the three Points *r*, *d*, and *s*, draw the Circle *c r d s*, it will be at Right  $\angle$ 's with the Oblique Circle *ZrN*, as was required:

### PROBLEM V.

*To set off any Number of Degrees upon a great Circle.*

In this Problem are three Cases.

**Case 1.** To set off any Number of Degrees upon the Primitive, suppose  $55^{\circ}.30'$ . from (*a*) upwards.

**RULE.** Take  $55^{\circ}.30'$ . from the Scale of Chords, and set from (*a*) to (*n*), and it's done.

**Case 2.** To set off any Number of Degrees upon a Right Circle.

**RULE.** This is best performed by a Scale of half Tangents, thus: Suppose it required to lay  $55^{\circ}.30'$ . from the Centre (*a*) towards *Z*.

Take

Take the half Tangent of  $55^{\circ}. 30'$ . and set from  $(a)$  to  $(x)$ , and it's done.

*Note*, If you set off any Number of Degr. upon a right Circle from the Centre, you must account from the Beginning of the half Tangents, and reckon forwards; but if you begin at the Primitive, you must also begin at  $90^{\circ}$  of the half Tangent, and reckon backwards; thus, if you had to set  $55^{\circ}. 30'$ . from  $Z$ , you must take  $55^{\circ}. 30'$ . backwards, viz. from  $90^{\circ}$ . to  $34^{\circ}. 30'$ . which will reach to  $(x)$ , as before.

Or it may be done by the Chords without a Line of half Tangents, in this manner: Take  $55^{\circ}. 30'$ . and set from  $H$  to  $(n)$ ; a Ruler laid from  $(n)$  to  $O$  will cut the right Circle  $Z \propto N$ , in the Point  $(x)$ , the same as before.

*Case 3.* To set off any Number of Degrees upon an Oblique Circle; suppose  $52^{\circ}. 30'$ .

**RULE.** Find the Pole of the given Oblique Circle; lay the Number of Degrees on the Primitive Circle, and reduce it from that to the Oblique Circle by the Help of its Pole. For Instance:

Lay a Ruler from  $Z$  to  $(r)$ , and it will cut the Primitive in  $(b)$ ; then from  $(b)$  set  $90^{\circ}$ . of the Chords to  $(y)$ , and lay a Ruler from  $(y)$  to  $Z$ , it will cut the Diameter  $H \propto O$  in  $(x)$ ; so is  $(x)$  the Pole of the Circle  $Z \propto N$ . Again, from  $O$  set  $52^{\circ}. 30'$ . of the Chords upwards to  $(d)$ , and lay a Ruler upon  $(x)$  and  $(d)$ , it shall cut the Oblique Circle in  $(s)$ ; so will  $r, n$  contain  $52^{\circ}. 30'$ . as was demanded.



## PROBLEM VI.

*To measure any Part of a great Circle.*

This Problem has likewise three Cases, which are but the Converse of those in the last; and therefore I think it needless to set them down, or give any Instances—For he who understands how to lay down any Quantity of Degrees upon any sort of Circle here mentioned, must of course know how to measure any Part of such Circle; only it may be observed in general.

1. That all Arches of the *Primitive* are measur'd by a Scale of *Chords*; as per Reverse to Case 1. Prob. the last.
2. That all Circles represented by *strait Lines*, are best measured by a Scale of *half Tangents*; as per Reverse to Case 2. Prob. the last, observing but the Directions there given. But if you have a *Sector* at hand, and know how to use it, you may with much Ease and Exactness measure any Part of a Right Circle, by considering it as a whole Line of Sines to a greater or a lesser Radius, and so set the Instrument accordingly.
3. That all *Oblique* Circles are measur'd from their Poles to the *Primitive*; and the Distance from their extreme Ends measured upon the *Chords*, will give their Quantities; as per Reverse to Case 3. Prob. ult.

## PROBLEM VII.

*To measure any Spherical Angle.*

In this Problem are four Varieties or Cases.

A Sphe-

A Spherical  $\angle$  is measured by the Arch of a great Circle, intercepted between the two containing Sides, the angular Point being the Pole of that Circle. See Def. 2. §. 1. of this Chap.

Or, the Distance of the Poles of the containing Sides, is equal to the Measure of the contained  $\angle$ . See the Rule to Case 4. Prob. 3.

And therefore the following Rule is universally useful in all the Cases of this Prob. (whether the Sides including the  $\angle$  be Arches of the Primitive, Right, or Oblique Circle) viz. Lay a Ruler from the  $\angle$  required to be known, over the Poles of the two Circles, including the said  $\angle$ , and observe at what two places the Ruler cuts the Primitive; for that Distance measured upon the Chords, gives the Quantity of the  $\angle$  sought.

*A general Rule in all the Varieties of this Problem, which is called reducing to the Primitive.*

*Example* It is required to find the Quantity of the  $\angle(r)$ , of the  $\Delta Z, r, s$ . Lay a Ruler from  $r$  to  $s$ , (the Pole of one of the Circles, including the  $\angle$ ) it cuts the Primitive in  $t$ ; and then a Ruler laid from  $r$  to  $n$ , (the Pole of the other Circle, including the  $\angle$ ) cuts the Primitive in  $u$ ; the Arch  $t, u$ , measured upon the Chords, gives the Quantity of the  $\angle$  required. I shall make no further mention of the Cases belonging to this Problem, seeing the Rule is general, and takes in all; only

*See Fig. following.*

*Note*, If the Poles fall both in one Diameter or Right Line, the Distance of those two Poles, measured upon a Scale of half Tangents, will give the Quantity of the  $\angle$ , without reducing to the Primitive Circle.



ment and Distance thereof from the Primitive, number'd backward) add on the Centre A, strike the Circle  $r, y, x$ , it shall be the Parallel required. *Note*, If you have not a Scale of half Tangents number'd upon your Rule, take half the Distance (*viz.*  $33^{\circ} 15'$ ) from your Scale of Tangents; which will perform the same thing. And so in any other Case whatever, the Line of half Tangents being only the Tangent of  $45^{\circ}$  double number'd; that is,  $5^{\circ}$  are called 10, Ten 20, Twenty 40, Forty 80, &c.

*Case 2.* To draw a Circle parallel to a Right Circle.

**RULE.** From the Chords lay the Parallel's Distance from the Right Circle; or the Complement thereof from the Pole of the Right Circle, both ways, and note those two Marks.

Then with the Tangent of the Parallel's Distance from the Pole of the Right Circle, and one Foot on each of those Marks, make an Arch to cut each other in the Centre of the Parallel required to be drawn.

*For Example:* Let it be required to draw a Parallel to the Right Circle HAO at  $30^{\circ}$  Distance.

Set  $30^{\circ}$  of the Chords from H and O to P and Q; or  $60^{\circ}$ , the Complement thereof, from Z to P and Q; then with the Tangent of  $60^{\circ}$ , and one Foot on P, describe an Arch over Z; with the same Distance, and one Foot on Q, cross the former Arch; which crossing is the Centre of the Parallel P  $m$  Q, required.

Or, if you have a Line of half Tangents on your Scale, you may find the Point ( $m$ ) through which the Parallel is to pass, and the Centre thereof, thus: Set the half Tangent of  $30^{\circ}$  from A to  $m$ ; then take the Tangent of  $60^{\circ}$ , (the Complement thereof) and set from  $m$ , upon the Dia-



meter  $NZ$ , extended, and where the Compass Point falls is the Centre, as before.

**Case 3.** To draw a Parallel to an Oblique Circle at any given Distance from it, suppose  $25^\circ$ .

**RULE.** Find the Pole of the given Oblique Circle  $Z, N$ , by *Prob. 2.* which is at  $(x)$ . Lay a Scale from  $Z$  to  $x$ , it will cut the Primitive in  $(d)$ . Set  $65^\circ$  (the Parallel's Distance from the Pole of the Oblique Circle) both ways from  $(d)$  to  $(l)$  and  $(p)$ , a Ruler laid from  $Z$  to  $(l)$  will cut  $HO$  in  $(b)$  (which may sometimes happen at the Centre.) Also a Ruler laid from  $Z$  to  $(p)$  will cut  $HO$  (continued) in  $R$ ; divide the Space  $Rb$  equally in  $(b)$ , and with the Extent  $bb$ , or  $bR$ , and one Foot in  $(b)$ , describe the Circle  $lhl$ , it is the Parallel demanded.

The Use of the preceding Problems in forming any Spherical Triangle, and measuring its Parts, shall be shown in the proper Places of the ensuing Work; and therefore I shall in the next place, here lay down and explain the *Universal*, or (as some call it) *Catholic* Proposition, before recommended, which is as follows.



## C H A P. IX.

*The Universal Proposition of the Lord  
Nepier, Baron of Merchiston in Scot-  
land; by which alone all the Cases of  
Spherical Triangles are resolved with  
great Ease and Facility.*

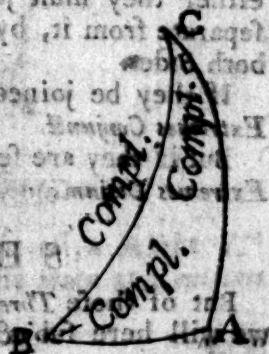
## S E C T. I.

**I**N every Rectangled Spherical Triangle, there  
are (besides the Right Angle) five other Parts;  
whereof those three which are more remote from  
the Right Angle, the Lord Nepier changed into  
their Complements.

As in the  $\triangle ABC$ , right  $\angle^a$  at A; for the three  
remote Parts, to wit, the  $\angle$ 's B and C, and the  
Hypotenuse, or Side BC, he takes their Comple-  
ments; but the Side AB and AC, being next to  
the right  $\angle$ , are not noted by their Complements.  
And these Parts thus noted, he termeth the five  
circular Parts of a right  $\angle^a \triangle$ :

Side A. B.  
Side A. C.  
Viz. the { Compl. of the  $\angle$  at B.  
          { Compl. of the  $\angle$  at C.  
          { Compl. of the Side BC.

The Right  $\angle$  being not  
looked upon as any Part in this  
Case; and therefore the Sides  
AB and AC, comprehending  
the Right  $\angle$  A, are supposed  
to be join'd together.



S E C T.

## S E C T. II.

Now in the Resolution of a Rectangled Spherical  $\Delta$ , there are two other Terms given (besides the Right  $\angle$ ) to find out a third.

## S E C T. III.

These three Terms (namely, two that are given, and the third which is required) must be first looked upon according to their circular Parts.

## S E C T. IV.

Of which, one is named the *Middle* (or mean) Part, the other two are call'd the *Extreme* Parts; borrowing the Appellation from the Situation of the Terms themselves: for of *three Terms*, one must necessarily be in the Middle, and the other *two* in the *Extremes*; therefore the circular Part of the middle Term is called the Middle Part, and the circular Parts of the extreme Terms are call'd the *Extreme Parts*.

## S E C T. V.

But the *Extreme Parts* may be twofold; for either they must join to the Middle Part, or be separate from it, by a Side or Angle interpos'd on both Sides.

If they be joined to it, then they are term'd *Extremes Conjunct.*

But if they are separate from it, they are call'd *Extremes Disjunct.*

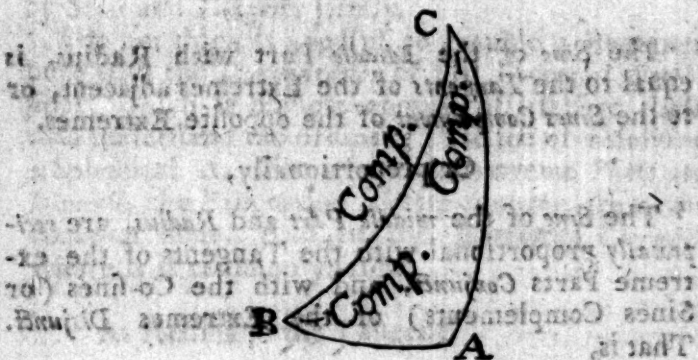
## S E C T. VI.

But of those *Three Terms* which fall in question, we will here subject all their Varieties in their circular

circular Parts, according as every one of them ought (in respect of each other) to be call'd the *Middle Part*, or the *Extremes Conjunct* or *Disjunct*, as in the following Analysis, or Table, is clearly demonstrated.

|        |   |          |                              |   |                      |   |                        |
|--------|---|----------|------------------------------|---|----------------------|---|------------------------|
| If the | { | Side A B | be the middle Part, then the | { | Side A C, & Compl. B | } | are Extremes Conjunct. |
|        |   | Side A C |                              |   | Side A B, & Compl. C |   |                        |
|        |   | Compl. C |                              |   | Side A C, & Com. B C |   |                        |
|        |   | Compl. B |                              |   | Side A B, & Com. B C |   |                        |
|        |   | Com. B C |                              |   | Com. B, & Com. C     |   |                        |

And { Compl. B C, and Compl. C  
Compl. B C, and Compl. B  
Side A B, and Compl. B  
Side A C, and Compl. C  
Side A B, and Side A C } are Extremes Disjunct.



## SECT. VII.

Therefore in the Resolution of a Right  $\angle^a$  Spherical Triangle, to know the *mean* and *extreme* Parts, you must observe, that

1. If



1. If one of the three *Terms* (which, besides the Right  $\angle$ , comes in question) doth stand alone by itself, severed from the other *Two* on both Sides, (as the Side BC, from the Sides AC and BA, by the  $\angle$  B and C-interposed) that shall be the *Middle Term*, and so its circular Part shall be called the *Middle Part*; and the other *Two* circular Parts are the *Extremes disjoined*. But,

2. If the three *Terms* do immediately adhere together, the *Middle Term* does easily shew the *Middle Part*, and the *Extreme Terms* the *Extreme Parts Conjunct*.

These Things being all rightly understood, the whole *Trigonometry of Sphericals* will be resolved by this one *Catholic or Universal Proposition*.

## S E C T. VIII.

### *The Universal Axiom.*

The *Sine* of the *Middle Part* with *Radius*, is equal to the *Tangents* of the *Extremes adjacent*, or to the *Sines Complement* of the opposite *Extremes*.

Or proportionally,

The *Sine* of the *middle Part* and *Radius*, are *reciprocally* proportional with the *Tangents* of the *extreme Parts Conjunct*, and with the *Co-sines* (or *Sines Complements*) of the *Extremes Disjunct*. That is,

As the *Radius*,

To the *Tangent* of one of the *Extremes Conjunct*,

So is the *Tangent* of the other *Extreme Conjunct*,

To the *Sine* of the *middle Part*; & *contra*.

Then

Then also,  
 As Radius,  
 To the Co-sine of one of the Extremes Disjunct,  
 So is the Co-sine of the other Extreme Disjunct,  
 To the Sine of the middle Part; & *contra*.

## COROLLARY.

Wherefore, 1. If the *middle Part* be sought, the *Radius* shall be in the first place of the Proportion; but if one of the Extremes be sought, then the other *Extreme* shall be in the first place. Of the second and third places, it mattereth nothing how they are disposed. Also,

2. If the *Extremes*, in any Proportion, be Disjunct from the *middle Part*, the Proportion will be perform'd by Sines only: But if the *Extremes* be Conjunct to the *middle Part*, it will be perform'd by Sines and Tangents jointly.

I do not think it needful to trouble a Beginner here with the Demonstration of this Universal Proposition, which is obvious enough of itself to those who understand the ordinary Practice of resolving a Spherical  $\Delta$ . For where the extreme Parts are Disjunct, the Proportion differs nothing from the common ones: And in the Extremes Conjunct, where it is commonly said,

As Radius to the Tangent,

We here say,

As the Co-Tangent to the Radius.

And likewise *Inversly* and *Contrarily*, which is plainly the same thing; because

The *Radius* is a mean Proportional, between the Tangent of an Arch, and the Tangent Complement of the same Arch.

But

But if any have a mind to see it demonstrated at large, let them turn to Mr. Norwood's Doctrine of Triangles, pag. 122. 6th Edit, where they may receive ample Satisfaction.

Before I conclude the Business of this Chapter, it will be needful to put the Learner in mind to observe, that when a Complement in any Proportion does chance to concur with a Complement in the circular Parts, you must always take the Sine itself, or the Tangent itself, instead of the Co-sine, or Co-tangent in the circular Parts; because the Co-sine of the Co-sine is the Sine itself, and the Co-tangent of the Co-tangent is the Tangent itself.

As in the first Case following, where BC is the middle Part, and the Complement of the  $\angle A$ , and Complement of AC, the *Extremes Disjunct*; here, because the two Extremes fall upon Complements in the circular Parts, therefore it must be the Sine of the  $\angle A$ , and the Sine of AC, because the Co-sine of the Complement is the Sine.

Or if this be not plain enough, take the two following general Directions:

1. If the *middle Part*, or either of the *Extremes Conjunct*, be the *Hypotenuse*, or either of the *Oblique  $\angle$ 's*; then instead of *Sine* and *Tangent*, you must say, *Sine-Complement*, and *Tangent-Complement*.

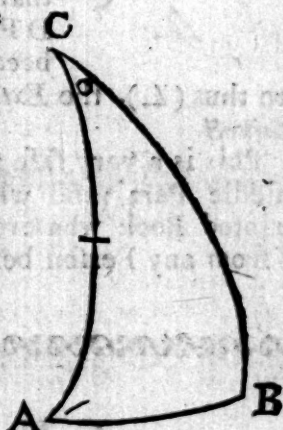
2. If either of the *Extremes Disjunct* be the *Hypotenuse*, or either of the *Oblique  $\angle$ 's*; instead of *Sine-Complement*, you must use the *Sine*.

In setting down this most famous and excellent Proposition, I have followed the Method, and generally the very Words of Mr. Leybourn, in his *Mathematical Institutions*, because I liked them better than my own. From which Book, and also Mr. Norwood's Doctrine of Triangles (before-mentioned) I do acknowledge to have received many useful Hints in this Part of Trigonometry, and to

have been very serviceable to me in composing much of what follows in the next Chapter : tho' a certain late Author, who transcribed a great Part of both into his *Posthumous Work*, (but indeed very inartificially, and to great Disadvantage, having swell'd into Words at length, what was there very plainly and comprehensively expressed by a few easy Symbols or Characters) and whose Performances have been recommended to the Publick by a late Reverend Person, was not so ingenuous as to own it.

*Since my writing this, an ingenious Acquaintance sent me the following easy Method for discovering the middle Part at any time by Numbers; thus,*

Suppose in the Triangle ABC, the things given and required be as they are there marked; begin to number from any Side or Angle, which you please; suppose at the Angle A, which is (1), the Side AC is (2), and the Angle C is (3); which place in a Triangular manner, as you see done in the Margin. Those Numbers that are close, (i. e.) not parted by any intervening Number, mark thus (—). But those which are parted, as the Number 1 and 3, which are separated by the Number (2) mark thus ( $\angle$ ). Observe what Number stands between the two like Characters, which here is the Number (2), or Side AC; which Side AC is the middle Part: And because the two



1 — 2

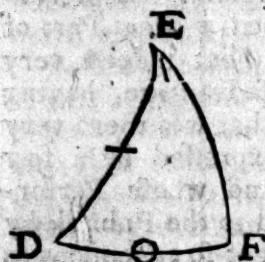
$\angle$  3  $\angle$

Q

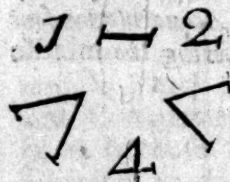
like



like Characters are close, (*i. e.*) mark'd thus (—) the *Extremes* A and C are *Conjunct*.



containing Sides, *viz.* DF and EF; therefore the



are thus ( $\angle$ ), the *Extremes* DE and Angle E are *Disjunct*.

This is a very safe and easy Way of finding the middle Part, and what I have not seen in any printed Book whatever, nor did I ever meet with it from any Person before.



## CHAP. X.

### *The Solution of the Sixteen Cases of Right-angled Spherical Triangles.*

HAVING already explained and demonstrated all things needful for the right understanding of Spherical Trigonometry in the foregoing

ing Sheets, I now come to give Directions for performing the several Cases belonging thereto, as follows.

But here it will be proper, first, to inform the Learner of the several things which may be given in the Resolution of a right  $\angle^d \Delta$ , which are in N<sup>o</sup>. 6. as underneath.

1. The Hypothenufe and an  $\angle$  adjoining being given; to find,

1. The Side opposite to the  $\angle$ .
2. The Side adjoining to the  $\angle$ .
3. The other Angle.

2. The Hypothenufe and a Side given; to find,

1. An  $\angle$  opposite.
2. The  $\angle$  adjoining.
3. The other Side.

3. A Side, and the  $\angle$  opposite, given; to find,

1. The other Side.
2. The other  $\angle$ .
3. The Hypothenufe.

4. A Side, and an  $\angle$  adjacent, given; to find,

1. The other Side.
2. The other  $\angle$ .
3. The Hypothenufe.

5. The two Sides given; to find,

1. Either of the Oblique  $\angle$ 's.
2. The Hypothenufe.

Q 2

6. The

6. The two Oblique  $\angle$ 's given; to find,

1. Either of the Sides.

2. The Hypothenufe.

In all Sixteen Cases; the Method of resolving which by the *Universal Axiom* afore-mentioned, we shall next declare.

\*\*\*\*\*

# S E C T. I.

## P R O P. I.

Containing Cases 1, 2, 3.

THE Hypothenufe AC, and an  $\angle$  adjoining,  
*viz.* A.

## C A S E I.

To find the Side opposite thereunto, BC, a  
*middle Part.*

Here the Part demanded is the *middle Part*, because separated from the Parts given by a Side and an  $\angle$ ; therefore the given Parts, *viz.* AC and  $\angle$  A are *Extremes Disjunct*; and because the Extremes fall upon Complements, the *Equation* by the *Universal Proposition* will be thus expressed:

$$\text{Sine } BC + \text{Rad.} = \text{Sine } AC + \text{Sine } \angle A.$$

*Example.* Let  $AC = 46^\circ 31'$ , and  $\angle A = 40^\circ$ .

To the Sine of AC, *viz.* — 9.8606821

Add the Sine of A — 9.8080675

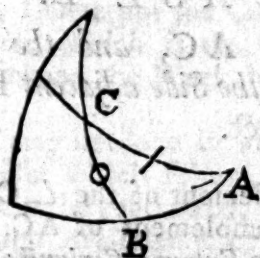
10

From their Sum abating Rad.

gives — 9.6687496

The Sine of  $27^\circ 48'$ , the Side BC required.

*This*



This, I suppose, to be very evident; for if the Logarithm Sine of BC, more Radius, be equal to the Sine of A, it must needs be a Truth, that if from the Aggregate, or Sum, of BC, and Radius, there be taken the Sine of AC, the Remainder will be the Sine of A. And so of all that follows.

Or proportionally by the latter Part of the Rule, thus:

| As Radius                                     | IO:             |
|-----------------------------------------------|-----------------|
| To the Sine of AC, $46^{\circ} 31'$ .         | 9.8606821       |
| So is the Sine of the $\angle A 40^{\circ}$ . | 9.8080675       |
| To the Sine of BC, $27^{\circ} 48'$ .         | <hr/> 9.6687496 |

*Instrumentally.*

Extend the Compasses from the Sine of  $90^{\circ}$  to  $46^{\circ} 31'$ , the same will reach from  $40^{\circ}$  to  $27^{\circ} 48'$ .

Note, BC, the Side required, will be less than a Quadrant, if the opposite  $\angle$  thereto be acute; but greater, if obtuse.

The Geometrical Resolution of the sixteen Cases of Right-angled Spherical Triangles, being comprehended under six Problems, I have reserved them for their proper Places.



CASE II.

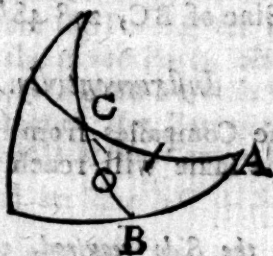
The Hypothenuſe AC, and the  $\angle C$ , being given, to find the Side adjacent BC, an Extreme Conjunct.

Here the Complement of the  $\angle C$  is the *middle Part*, and the Complement of AC, and the Side BC, required, are *Extremes Conjunct*; therefore the Equation by the *Universal Axiom* may be thus expreſs'd.

Co-line  $\angle C + \text{Rad.} = \text{Tang. BC} + \text{Co-tang. AC.}$

*Example.* Let  $AC = 46^{\circ} 31'$ , and  $\angle C = 60^{\circ}$ .  
To the Co-line of the middle Part  $C. 60^{\circ} 00'$ . 9.698970  
Add Radius 10.

9.698970  
From their Sum ſubtract *ct. AC*  $46^{\circ} 31'$ . 9.976997  
Remains the Tangent of  $27^{\circ} 48'$ . 9.721973  
The Side BC required.



Or Proportionally.

As *ct. of AC*  $46^{\circ} 31'$ . 10.023003  
To Radius 10.  
So *ct.  $\angle C$*   $60^{\circ} 00'$  9.698970  
To Tangent of BC,  $27^{\circ} 48'$ . 9.721973

Note

*Note* here, instead of the Logarithm Co-tangent of  $46^{\circ} 31'$ , I have put the Complement Arithmetical thereof; which is the Tangent of  $46^{\circ} 31'$ , and so performed the Work by *Addition* instead of *Subtraction*.

The like may be done in any of the rest of the fourteen Cases which follow, when Radius is not the Number to be subtracted, whether you perform the Operation *Analogically*, or by the *Equation*, as above; which is very evident, if in the room of the Logarithm Co-tangent of A C,  $46^{\circ} 31'$  (*viz.* 9.976997) you place its Complement Arithmetical, *viz.* 10.023003, and add the Logarithms together, rejecting the Fig. in Tens Place of the Index; which will produce the same Answer as before.

The Manner of finding the *Arithmetical Complement* you have in *Page 89.* with some farther Directions in this matter, and therefore need not be here repeated.

*To perform this Case Instrumentally.*

Extend the Compasses from  $90^{\circ}$  to  $30^{\circ}$  (the Complement of the  $\angle C$ ) in the Sines, apply that Extent from 45 in the Line of Tangent downwards, keeping the lower Leg of the Compasses fix'd, till you pull in the other to  $43^{\circ} 29'$ . that last Extent will reach from  $45^{\circ}$  to  $27^{\circ} 48'$ , the Side required.

Or else you may use *Gross-work*, and extend from the Co-tangent of A C to the Radius, or Sine of  $90^{\circ}$ , that Extent will reach from the Co-sine of the  $\angle C$ , to the Tangent of B C; which Way of working, in many Cases, is preferable to the other; especially when the Extent is too large for the Compasses, and therefore shou'd be carefully heeded by a Beginner.

So the Extent from  $43^{\circ} 29'$  on the Tangent to  $90^{\circ}$  on the Sines, will reach from  $30^{\circ}$  on the Sines to  $27^{\circ} 48'$  on the Tangents, the same as above.

*Note,*

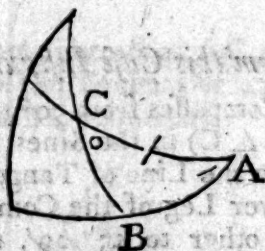
Note, When each given thing is less or more than  $90^\circ$ , the Side sought is less than  $90^\circ$ . But if one is less, and the other more, the Side sought is more than  $90^\circ$ .

### C A S E III.

The Hypothenuſe AC, and the  $\angle A$  given, to find the other Oblique  $\angle C$ , an Extreme Conjunct.

Here the Hypothenuſe is the middle Part, becauſe the Parts given and Part demanded lie together, and are all of them noted with their Complements; therefore the Equation will be as follows:

$$\text{Co-ſine AC} + \text{Radius} = \text{ct. } \angle A + \text{ct. } \angle C.$$



*Example.* Let AC be  $46^\circ 31'$  (as before) and the  $\angle A$   $40^\circ 00'$ .

To the *cs.* of the middle Part AC  $46^\circ 31'$ . 9.837679

Add Radius ————— 10.

From their Sum ſubtract *ct.*  $\angle A$   $40^\circ 00'$  } 9.923814  
 $00'$ , (or add the Co. Ar.) ————

Gives the Co-tang. of the  $\angle C$ ,  $60^\circ 00'$ . 9.761493

Or

*Or Proportionally.*

As *ct.*  $\angle A$ ,  $40^{\circ} 00'$  ——— *Co. Ar.* ——— 9.923814  
 To *cs.*  $AC$   $46^{\circ} 31'$  ——— ——— ——— 9.837679  
 So is Radius ——— ——— ——— 10.  
 To *ct.*  $\angle C$ ,  $60^{\circ} 00'$  ——— ——— ——— 9.761493

These three Examples to the foregoing Cases being wrought thus at large, will, I am persuaded, be sufficient Instructions to the meanest Capacity for ordering the Logarithms, and performing the Operation of those which remain, either way; and therefore I shall forbear putting down the Logarithms to the Examples any more as I go along; and only give Directions, both ways, according to the *Universal Rule*, for placing the Terms of the Question in their proper Order, and set down the Answers: But because some may judge it needful to have them somewhere in the Book, that the Learner should not be at a loss for any thing, supposing him not to have a Table of Logarithms at hand, I have for this Reason plac'd them in two Tables, one for Right, and t'other for Oblique Sphericks at the End of Trigonometry.

And altho' either of these Methods of resolving a Spherical  $\Delta$  are equally true and right, and may be practised by a Beginner according as he likes best; yet I shall take the Liberty to recommend the first, (notwithstanding the latter is most frequently used) as more easy and natural to the Apprehension, and less burthensome to the Memories of young Practitioners.

*To perform this Case Instrumentally.*

Extend the Compasses from  $43^{\circ} 29'$  (the Complement of  $AC$ ) to  $90^{\circ}$  of the Sines, the same  
 Extent



Extent will reach from  $50^{\circ} 00'$  † (the Complement of the  $\angle A$ ) to  $30^{\circ}$  in the Tangents, the Comp. of  $\angle A = 60^{\circ}$ .

*Note, When each given thing is less or more than  $90^{\circ}$ , the  $\angle$  sought is Acute; but when one is more, and the other less, it is Obtuse.*

The GEOMETRICAL Solution of the three foregoing Cases, in each of which there are given

The Hypothenuse  $AC\ 46^{\circ} 31'$  (\*) and one of the Oblique  $\angle$ 's, viz.  $A = 40^{\circ} 00'$ , to find

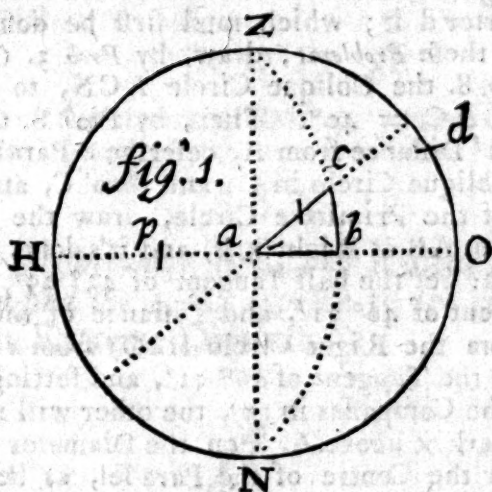
1. The opposite Side
  2. The Side adjacent, viz.
  3. The other Oblique  $\angle$
- $\left\{ \begin{array}{l} BC. \\ AB. \\ C. \end{array} \right.$

\* *Note, The Right  $\angle$  is always supposed to be one of the given things.*

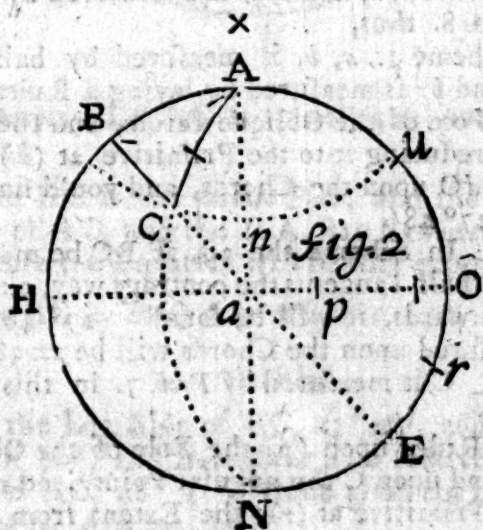
First, To make the  $\Delta$  so that the given  $\angle BAC$  may be at the Centre of the Primitive Circle.

The Primitive being drawn and quarter'd as before directed, make the  $\angle BAC = 40^{\circ} 00'$ , by Prob. 3. Case 1. Chap. 8. 2dly, Make  $AC = 46^{\circ} 31'$ , by Prob. 5. Case 2. That is, Set the half Tangent of  $46^{\circ} 31'$  from A to C, and thro' the Point C, and at Right  $\angle$ 's with AB draw an Oblique Circle, and it's done.

† Here it ought to be observed, that the Line of Tangents is supposed to be continued no further than  $45^{\circ}$ ; and so number'd back again, as most Lines of Tangents are, viz.  $40^{\circ}$  is reckon'd 50,  $30^{\circ} 60$ ,  $20^{\circ} 70$ , &c.



2. To make the  $\Delta$  so that the given  $\angle BAC$ , may be at the Primitive Circle.



Having

Having drawn the Primitive Circle  $AHNO$ , and quarter'd it; which must first be done, in every of these *Problems*; draw, by *Prob. 3. Case 2. §. 3. Chap. 8.* the Oblique Circle  $ACN$ , to make the  $\angle BAC = 40^\circ$ . Then, by *Prob. 8. Case 2.* at  $46^\circ 31'$  Distance from  $A$ , describe a Parallel to cut the Oblique Circle in  $C$ ; and thro'  $C$ , and the Centre of the Primitive Circle, draw the Line  $BE$ , to cut  $AB$  at Right  $\angle$ 's, and it's done.

Or thus: Set the half Tangent of  $43^\circ 29'$ , (the Complement of  $46^\circ 31'$ , and Distance of the Parallel from the Right Circle  $HAO$ ) from  $a$  to  $n$ ; then take the Tangent of  $46^\circ 31'$ , and setting one Foot of the Compasses in  $(n)$ , the other will reach to this Mark  $\times$  above  $A$ , (on the Diameter  $AN$  extended) the Centre of the Parallel, as before. But now

*To measure the Things required.*

The Sides  $a, b$ , and  $b, c$ , are measured by *Prob. 5. § 3. Chap. 8.* thus,

In Scheme 1.  $a, b$ , is measured by half Tangents, and  $bc$  is measured by laying a Ruler upon  $(p)$  the Pole of the Oblique Circle, and the Point  $(c)$ , and reducing it to the Primitive at  $(d)$ ; then measure  $dO$  upon the Chords, and you'll find it to contain  $27^\circ 48'$ .

Again: In Scheme the 2d, if  $BC$  be measured upon the half Tangents the contrary way, *viz.* from  $90^\circ$  downwards, it will be found  $= 27^\circ 48'$ ; and  $AB$  measured upon the Chords will be  $= 38^\circ 56'$ .

The  $\angle C$  is measured by *Prob. 7.* in this manner:

Lay a Ruler upon  $(p)$  the Pole of the Oblique Circle, and upon  $C$  the angular Point, and reduce it to the Primitive at  $(r)$ ; the Extent from  $(r)$  to  $(u)$  (the Pole of the Right Circle  $BE$ ) measured upon the Chords, is  $= 60^\circ$ , the Quantity of the  $\angle C$ .

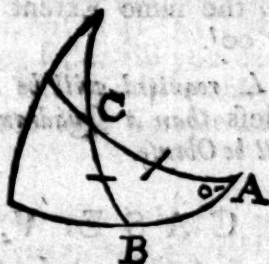
$\angle C$ . In the same Method is found the  $\angle C$  in Scheme 1.

PROP. II.

Containing Case 4, 5, and 6.

CASE IV.

The Hypothenuse AC, and a Side, viz. BC, to find the  $\angle A$ , opposite to the given Side; an Extreme Disjunct.



Example. Let AC be  $= 46^{\circ} 31'$ , and BC  $= 27^{\circ} 48'$ .

Here BC is the *middle Part*, and the Complements of AC, and the  $\angle A$ , are *Extremes Disjunct*; therefore the Equation will be thus expressed:

$$\text{Sine BC} + \text{Rad.} = s. AC + s. \angle A.$$

That is,

To the Log. Sine of BC,  $27^{\circ} 48'$ , add Rad. (or Sine of  $90^{\circ}$ ) and from their Sum subtract the Log. Sine of AC,  $46^{\circ} 31'$ , which gives the Logarithm Sine of  $\angle A = 40^{\circ} 00'$ .

R

This



This requires no more Words to make it easy and intelligible to the meanest Apprehension. But if you had rather use Addition than Subtraction, you may put the Arithm. Complement of BC, instead of the Sine thereof, and so perform the Work by Addition only, as in the last foregoing Case.

*Or Proportionally.*

As  $\text{AC } 46^{\circ} 31'$  : Rad ::  $\text{BC } 27^{\circ} 48'$  :  $\angle \text{A } 40^{\circ} 0'$

*Instrumentally.*

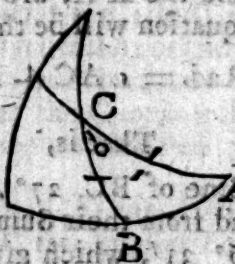
Extend from  $46^{\circ} 31'$  to Rad. or  $90^{\circ}$ , on the Line of Sines, the same Extent will reach from  $27^{\circ} 48'$  to  $40^{\circ} 00'$ .

Note, The  $\angle$  required will be Acute, if the Side given be less than a Quadrant; but if greater, then it will be Obtuse.

## CASE V.

*The Hypothenuse AC, and the Side BC, given, to find the adjacent  $\angle \text{C}$ , the middle Part.*

Example, Let AC be  $= 46^{\circ} 31'$ , and BC  $= 27^{\circ} 48'$ .



In this Case the Complement of the  $\angle \text{C}$  is the middle Part, and the Complement of AC and BC are

are *Extremes Conjunct*. Ergo, the Equation is as follows:

$$ct. AC 46^{\circ} 31' + t. BC 27^{\circ} 48' = \dagger Rad. + cs. \angle C, 60^{\circ} 00'.$$

*Or Proportionally.*

$$As Rad : ct. AC 46^{\circ} 31' :: t. BC 27^{\circ} 48' : cs. \angle C = 60^{\circ}.$$

*Instrumentally.*

Set the Compasses from  $45^{\circ}$  to  $27^{\circ} 48'$  on the Line of Tangents, and apply that Extent from  $43^{\circ} 29'$  (the Complement of  $46^{\circ} 31'$ ) backwards, keeping the lower Leg fix'd, till you extend the other to  $45^{\circ}$ ; this last Extent will reach from the Sine of  $90^{\circ}$  to  $30^{\circ}$ , the Sine Complement of the  $\angle C = 60^{\circ}$ .

Note, When each given thing is more or less than  $90^{\circ}$ , the  $\angle$  demanded is always less than a Quadrant; but if one be less, and the other more, it's greater than a Quadrant.

† I have here, and in some other of the Cases which follow, changed the Sides of the Equation, putting for the two foremost Terms, those, which, in Strictness to the Rule, ought to be the two last Terms, only for Order Sake, which I mention that the Learner may not stumble at it.

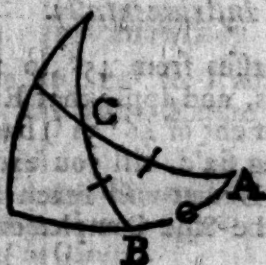
R<sub>2</sub>

CASE

CASE VI.

*The Hypotenuse AC, and the Side BC, given, to find the other Side AB, an Extreme Disjunct.*

*Exam.* Let  $AC = 46^{\circ} 31'$ , and  $BC = 27^{\circ} 48'$ .



Here the Complement of AC is the *middle Part*, and AB and BC are *Extremes Disjunct*; therefore the Equation will be thus:

$$\text{cs. } AC \ 46^{\circ} 31' + \text{Rad.} = \text{cs. } BC \ 27^{\circ} 48' + \text{cs. } AB \ 38^{\circ} 56'$$

*Or Proportionally.*

$$\text{As cs. } BC \ 27^{\circ} 31' : \text{cs. } AC \ 46^{\circ} 31' :: \text{Rad} : \text{cs. } AB = 38^{\circ} 56'$$

*Instrumentally.*

Extend from  $62^{\circ} 12'$  (the Complement of BC) to  $43^{\circ} 29'$  (the Complement of AC) the same Extent of the Compasses will reach from the Sine of  $90^{\circ}$  to  $51^{\circ} 04'$ , the Sine Complement of AB =  $38^{\circ} 56'$ .

*Note, If the Things given be both of one kind; that is, both less or more than a Quadrant, the Side required*

quired is less than  $90^\circ$ . But if they are of different kinds, i. e. one less, and the other more, the Side sought is more than  $90^\circ$ .

The GEOMETRICAL Solution of these three Cases; in each of which there are given,

The Hypotenuse A C,  $46^{\circ} 31'$ .

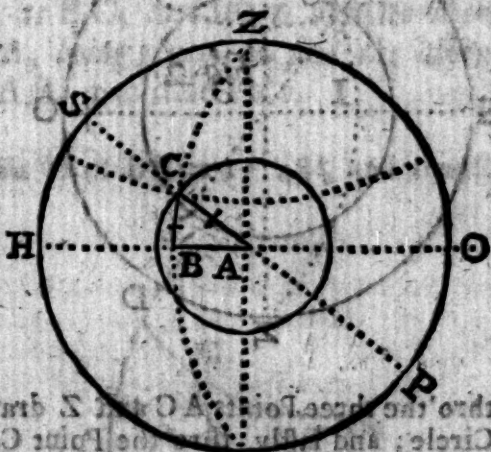
and a Side, viz. BC,  $27^{\circ} 48'$ . to find

1. The  $\angle$  opposite to the given Side }  
 2. The  $\angle$  adjoining \_\_\_\_\_ } viz. { A.  
 3. The other Side \_\_\_\_\_ } C.  
 A B.

1. To make the  $\Delta$  so, that one of the Oblique  $\angle$ 's may be at the Centre of the Primitive Circle.

By *Prob 8. §. 3. Chap. 8.* Draw a Circle parallel to the Primitive at the Distance of  $AC, 46^{\circ} 31'$  from the Pole A; that is, with the half Tangent  $46^{\circ} 31'$ , and one Foot on A describe a Circle.

Again, by *Prob. 8.* draw a Circle parallel to HO, at the Distance of BC  $27^{\circ} 48'$ , which cuts the other parallel in C.



**R 3**

## Then

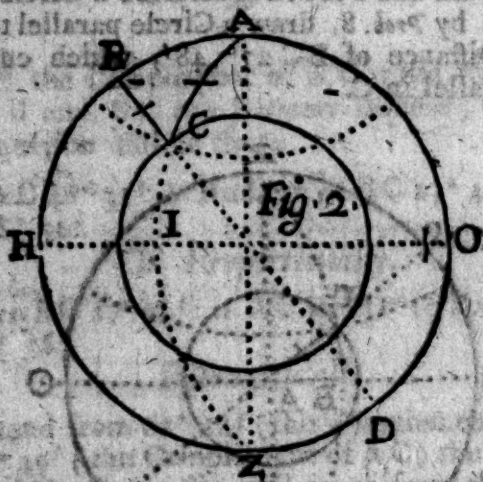


Then (by *Prob. 18. Chap. 2.*) thro' the three Points *Z, C, N*, draw an Oblique Circle at right  $\angle$ 's to *AB*, whose Centre will be in the Diameter *HO* extended, and may be easily found with a very few Trials.

Lastly, Thro' *C* and the Centre of the Primitive Circle *A*, draw the Right Circle *SP*, and it's finished.

2. To make the  $\Delta$ , that one of its Oblique  $\angle$ 's may be at the Primitive Circle.

Draw a Circle parallel to the Right Circle *HO*, by *Prob. 8.* at  $46^{\circ} 31'$  distant from *A*, its Pole, then (by *Prob. the same*) draw a Circle parallel to the Primitive at the Distance of *BC*  $27^{\circ} 49'$  from it, which will cut the other in *C*.



Then thro' the three Points *A C* and *Z* draw an Oblique Circle; and lastly, thro' the Point *C*, and the Centre of the Primitive, draw the Diameter *BD*, and it is done.

*To measure the Things required.*

The Side AB, in Fig. 1. is measured by half Tangents, as in the last Cases; and in the second Fig. it is measured by the Chords.

The Measure of the  $\angle BAC$  in Fig. (1.) is SH, measured upon the Chords; and the Measure of  $BAC$ , Fig. 2. is HI, measured upon the Scale of half Tangents from  $90^\circ$  downwards; and the  $\angle C$  is measured by Prob. 7. See the Geometrical Solution of the last three Cases.

So the Quantity of the Side AB will be found:  $= 38^\circ 56'$ , the  $\angle A = 40^\circ$ , and the  $\angle C = 60^\circ$ .

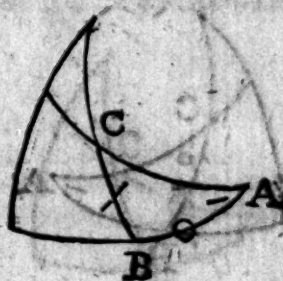
P R O P. III.

Containing Case 7, 8, and 9.

C A S E VII.

*A Side, as BC, and the  $\angle$  opposite thereunto, viz. A, being given, to find the other Side, viz. AB, a middle Part.*

*Example.* Let  $BC = 27^\circ 48'$ , and the  $\angle A = 40^\circ 00'$ .



Here

Here you see the Term demanded, viz. the Side AB, is the middle Part; and the Complement of the  $\angle A$ , and the Side BC, are *Extremes Conjunct*; therefore the Equation will be,

$$\therefore BC 27^{\circ} 48' + ct. \angle A 40^{\circ} 00' = Rad. + s. AB 38^{\circ} 56'.$$

*Or Proportionally.*

$$As Rad : ct. \angle A 40^{\circ} 00' :: s. BC 27^{\circ} 48' : s. AB = 38^{\circ} 56'.$$

*Instrumentally.*

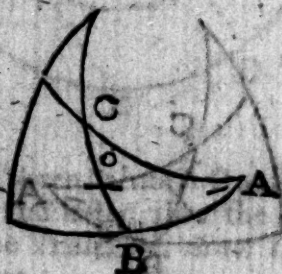
Extend from  $50^{\circ}$  to  $27^{\circ} 48'$  in the Tangents, the same Extent will reach from  $90^{\circ}$  to  $38^{\circ} 56'$  in the Line of Sines.

Note, If the  $\angle$  sought be Acute, or Obtuse, the Side required is less or more than a Quadrant. Or the Hypotenuse and given Side each less or more than  $90^{\circ}$ , the required Side is less than a Quadrant; but if one be more, and the other less, it's more than  $90^{\circ}$ .

### C A S E VIII.

BC and the  $\angle$  opposite, viz. A, to find the other oblique  $\angle C$ ; an *Extreme Disjunct*.

*Example.* Let the Side BC =  $27^{\circ} 48'$ , and  $\angle A = 40^{\circ}$ .



In

In this Case the Complement of the  $\angle A$  is the middle Part, and the Side BC, and the Complement of the  $\angle C$ , are *Extremes Disjunct*; therefore the Equation is

$$\text{cs. } \angle A 40^\circ + \text{Rad.} = \text{cs. BC } 27^\circ 48' + \text{s. } \angle C 60^\circ.$$

And the *Analogy*,

$$\text{As cs. BC } 27^\circ 48' : \text{Rad.} :: \text{cs. } \angle A 40^\circ : \text{s. } \angle C = 60$$

Or *Instrumentally*.

Extend from  $62^\circ 12'$  (the Complement of  $27^\circ 48'$ ) to Rad. on the Sines, the same Extent being applied from  $30^\circ$  (the Compl. of  $40^\circ$ ) will reach to  $60^\circ$ .

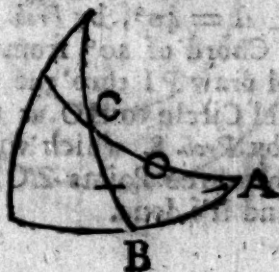
Note, When the Hypotenuse and given  $\angle$  are each of them less or more than a Quadrant, the required  $\angle$  is Acute; but if one be more, and the other less, it is Obtuse.

Or, if the Side adjoining to the given  $\angle$  be less, or more than a Quadrant, the required  $\angle$  is accordingly less or more than  $90^\circ$ .

### CASE IX.

BC and the  $\angle A$ , opposite therunto, being given, to find the Hypotenuse AC, an Extreme Disjunct.

Example. Let  $\text{BC} = 27^\circ 48'$ , and  $\angle A = 40^\circ$ .



Here



Here the Side BC is the middle Part, and Complement of the  $\angle A$ , and Compl. of AC, are *Extremes Disjunct*; therefore the Equation will be as follows :

$$s. BC 27^{\circ} 48' + \text{Rad.} = s. \angle A 40^{\circ} + s. AC 46^{\circ} 31'.$$

And the *Analogy*,

$$\text{As } s. \angle A 40^{\circ} : s. BC 27^{\circ} 48' :: \text{Rad.} : s. AC = 46^{\circ} 31'.$$

Or *Instrumentally*.

Extend from  $40^{\circ}$  to  $27^{\circ} 48'$  on the Sines, that Extent will reach from  $90^{\circ}$  to  $46^{\circ} 31'$ .

Note, If both the Sides, or both  $\angle$ 's, be of one kind, the Hypotenuse is less than a Quadrant; but if they be of different kinds, it's more than a Quadrant.

To solve the three foregoing Cases Geometrically.

In each of which there is given

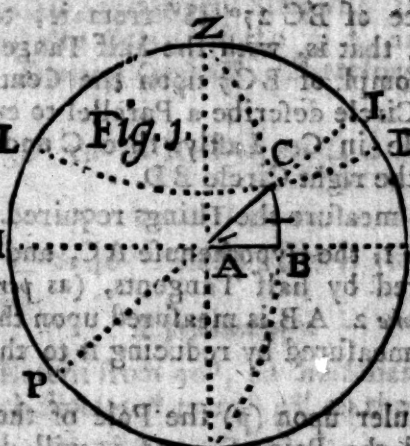
The Side BC =  $27^{\circ} 48'$ , and  $\angle A = 40^{\circ}$ ,  
to find

- |                         |         |     |
|-------------------------|---------|-----|
| 1. The other Side —     | } oiz { | AB. |
| 2. The other $\angle$ — |         | C.  |
| 3. The Hypotenuse —     |         | AC. |

1. To delineate the  $\Delta$ , that one of its Oblique  $\angle$ 's may be at the Primitive Circles Centre.

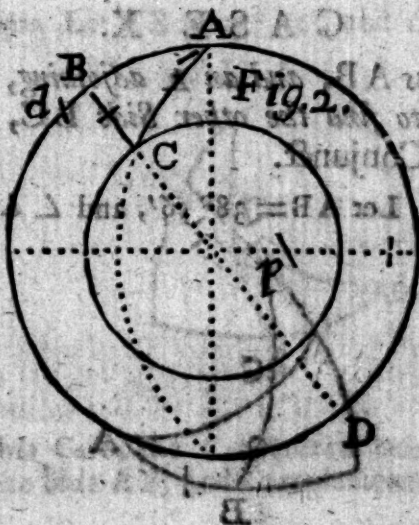
Make the  $\angle A = 40^{\circ}$ , by *Prob. 3. Case 1. Ch. 8.* thus: Set the Chord of  $40^{\circ}$  from O to I on the Primitive, and draw PI thro' the Centre A; then draw a parallel Circle to HO at the Distance of BC  $27^{\circ} 48'$ , by *Prob. 8.* which will cut PI in C. Lastly, thro' the three Points ZCN draw an Oblique Circle, and it's done.

2. To



2. To delineate the  $\Delta$ , that the given  $\angle$  may be at the Primitive Circle.

Draw the Oblique Circle ZCN, making an  $\angle$  with the Primitive at A,  $= 40^\circ$ . by Prob. 3. Case 2. Then describe a Circle parallel to the Primitive at



the Distance of BC  $27^{\circ} 48'$  from it; to cut the other in C, that is, with the half Tangent of  $62^{\circ} 12'$  (the Compl. of BC) upon the Centre of the Primitive Circle describe a Parallel to cut the Oblique Circle in C. Lastly, thro' C and the Centre, draw the right Circle BD.

### To measure the Things required.

In *Scheme 1.* the Hypothenuse  $A\hat{C}$ , and Side  $BA$ , are measured by half Tangents, (as *per Prob. 6.*) But in *Scheme 2.*  $AB$  is measured upon the Chords, and  $AC$  is measured by reducing it to the Primitive, thus:

Lay a Ruler upon (*p*) the Pole of the Oblique Circle, and the Point C, and it will divide the Primitive Circle in (*d*); so is *dA* applied to the Chords, the Measure of the Hypotenuse AC =  $46^{\circ} 31'$ . See *Prob. 5. Case 3. Ch. 8.* The  $\angle C$  is measured by *Prob. 7.*

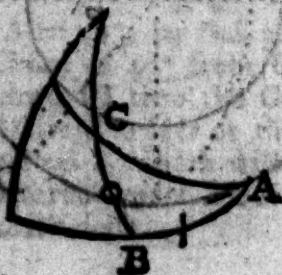
## PROP. IV.

Containing Cafe 10, 11, and 12.

## CASE X.

*A Side, as AB, and an  $\angle$  adjoining, viz. A, given, to find the other Side BC, an Extreme Conjunct.*

**Example.** Let  $AB = 38^{\circ} 56'$ , and  $\angle A = 40^{\circ}$ .



In

In this Case, the given Side AB is the *middle Part*; and the Comp. of the  $\angle A$ , and the Side BC, are *Extremes Conjunct.* Therefore the Equation is,

$$s. AB 38^{\circ} 56' + \text{Rad.} = ct. \angle A 40^{\circ} + s. BC 27^{\circ} 48'.$$

Or Analogically.

$$\text{As } ct. \angle A 40^{\circ} : \text{Rad.} :: s. AB 38^{\circ} 56' : s. BC = 27^{\circ} 48'.$$

Instrumentally.

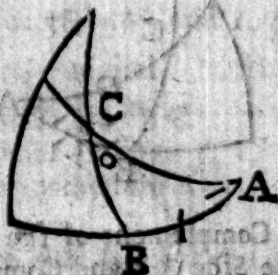
The Extent from  $90^{\circ}$ , on the Sines, to  $38^{\circ} 56'$ , will reach from  $50^{\circ}$  to  $27^{\circ} 48'$  on the Tangents.

Note, The Side required will be less than  $90^{\circ}$ , if the given  $\angle$  be Acute; but greater, if Obruse.

## CASE II.

AB, and the  $\angle A$  adjacent, given, to find the other Oblique  $\angle C$ , a middle Part.

Example. Let  $AB = 38^{\circ} 56'$ , and  $\angle A = 40^{\circ}$ .



In this Case, the  $\angle$  required being the *middle Part*, the Side AB, and Complement of the  $\angle C$ ,  
S are



are *Extremes Disjunct*, and so the Equation will be as follows:

$$cs. AB 38^{\circ} 56' + s. \angle A 40^{\circ} = Rad. + cs. \angle C 60^{\circ}.$$

And the *Analogy*.

$$As Rad. cs. AB 38^{\circ} 56' :: s. \angle A 40^{\circ} : cs. \angle C = 60^{\circ}.$$

Or *Instrumentally*, thus:

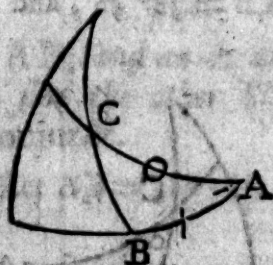
Extend from  $90^{\circ}$  to  $51^{\circ} 04'$  on the Sines, that Extent will reach from  $40^{\circ}$  to  $30^{\circ}$ , the Comp. of the  $\angle C = 60$ .

Note, The  $\angle$  found will be Acute, if the Side given be less than a Quadrant; Otruse, if greater.

## C A S E XII.

AB and the adjacent  $\angle A$ , to find the Hypothenuse AC, an *Extreme Conjunct*.

Example. Let  $AB = 38^{\circ} 56'$ , and  $\angle A = 40^{\circ}$ .



Here the Complement of the  $\angle A$  is the *middle Part*, and the Side AB and Comp. of AC are *Extremes Conjunct*. Therefore it will be by the Equation,

$$cs. \angle A 40^{\circ} + Rad. = s. AB 38^{\circ} 56' + ct. AC 46^{\circ} 31'.$$

And

And *Analogically*.

As  $t. AB\ 38^{\circ}\ 56'$  : Rad ::  $cs. \angle A\ 40^{\circ}$  :  $ct. AC$   
 $= 46^{\circ}\ 31'$ .

Or *Instrumentally*.

Extend from  $90^{\circ}$  in the Sines to  $50^{\circ}$  (the Comp. of the  $\angle A$ ) that Extent will reach from  $45^{\circ}$  to  $52^{\circ}\ 30'$  in the Tangents: Keep the lower Leg fix'd, till you pull in the other to  $51^{\circ}\ 04'$  (the Compl. of  $AB$ ) the last Extent will reach from  $45^{\circ}$  to  $46^{\circ}\ 31'$ .

Or more easily as before, in *Case 2*.

Extend from the Tangent of  $38^{\circ}\ 56'$  to the Sine of  $90^{\circ}$ , that Extent will reach from the Sine of  $50^{\circ}$  to  $46^{\circ}\ 31'$  in the Tangents.

*Note, Each given thing less or more than  $90^{\circ}$ , the Hypothenufe is less than a Quadrant; but greater, if they are of a different Affection.*

The GEOMETRICAL Effect of each of these Cases, in which there are given

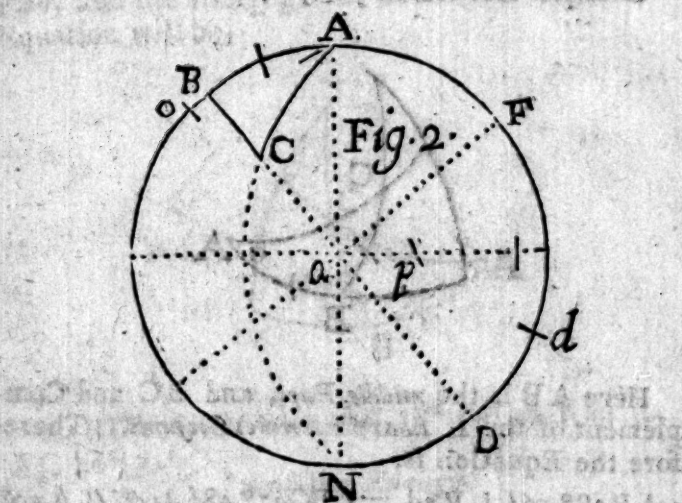
The Side  $AB = 38^{\circ}\ 56'$ , and  $\angle A = 40^{\circ}$ .

to find

1. The other Side  $BC$ .
2. The other Oblique  $\angle C$ .
3. The Hypothenufe  $AC$ .

1. To delineate the  $\Delta$ , that the given  $\angle$  may fall at the Centre of the Primitive Circle.





The Things required are thus found:

**Fig. 1.** AC is measured upon the half Tangents and BC and the  $\angle C$  are measured by reducing to the Primitive Circle, as before directed; so the Measure of BC is the Arch Hd =  $27^{\circ} 48'$ , and the Measure of the  $\angle C$  is the Arch Fe =  $60^{\circ}$ .

But in *Scheme 2*. BC is measured upon the ball Tangents from  $90^\circ$  downwards, and the Arch  $A = 46^\circ 31'$ , and  $Fd = 60^\circ$ , are the Measures of AC and the  $\angle C$ , being reduced to the Primitive See the Directions of Cases 1, 2, 3.

PROP. V.

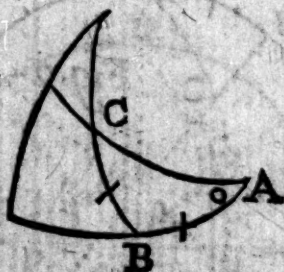
Containing Case 13 and 14.

### CASE XIII.

The two Sides  $AB$  and  $BC$  given, to find either of the Oblique  $\angle$ 's, suppose  $A$ , an Extrem Conjunct.



*Example.* Let  $AB = 38^{\circ} 56'$ , and  $BC = 27^{\circ} 48'$ .



Here  $AB$  is the *middle Part*, and  $BC$  and Complement of the  $\angle A$  are *Extremes Conjunct*. Therefore the Equation is,

$$s. AB 38^{\circ} 56' + \text{Rad.} = s. BC 27^{\circ} 48' + ct. \angle A 40^{\circ}.$$

And the *Analogy*.

$$\text{As } s. BC 27^{\circ} 48' : s. AB 38^{\circ} 56' :: \text{Rad} : ct. \angle A = 40^{\circ}.$$

*Instrumentally.*

Extend from  $38^{\circ} 56'$  to  $90^{\circ}$  on the Sines, the same Extent will reach from  $27^{\circ} 48'$  to  $40^{\circ}$  on the Tangents.

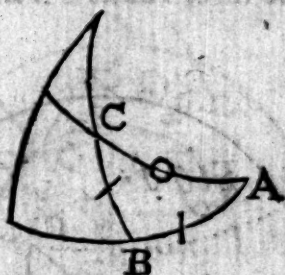
*Note, The  $\angle$  sought will be Acute, if the Side opposite to the required  $\angle$  be less than a Quadrant; but Obtuse, if greater.*

#### CASE XIV.

$AB$  and  $BC$  given, to find the Hypotenuse  $AC$ , the middle Part.

*Exam.* Let  $AB = 38^{\circ} 56'$ , and  $BC = 27^{\circ} 48'$ .

In this Case, what's required being the *middle Part*, and the things given, *Extremes Disjunct*; the Equation will be,



$$\text{cs. } BC \ 27^{\circ} \ 48' + \text{cs. } AB \ 38^{\circ} \ 56' = \text{Rad.} + \text{cs. } AC \ 46^{\circ} \ 31'.$$

And the *Analogy*,

$$\text{As Rad} : \text{cs. } BC \ 27^{\circ} \ 48' :: \text{cs. } AB \ 38^{\circ} \ 56' : \text{cs. } AC = 46^{\circ} \ 31'.$$

*Instrumentally.*

Extend from  $90^{\circ}$  to  $62^{\circ} \ 12'$  (the Compl. of BC) the same will reach from  $51^{\circ} \ 04'$  (the Compl. of AB) to the Compl. of  $AC = 43^{\circ} \ 29'$ .

If the Sides be of the same Affection, the Hypothennuse is less than  $90^{\circ}$ ; but if they be of a different Affection, it is greater than a *Quadrant*, or  $90^{\circ}$ .

The GEOMETRICAL Effect of the two Cases above. In either of which there is given

The Sides  $AB = 38^{\circ} \ 56'$ , and  $BC = 27^{\circ} \ 48'$ .

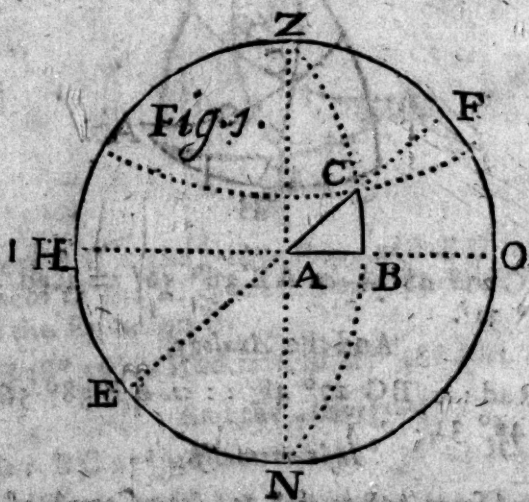
to find,

1. The Oblique  $\angle A$ .
2. The Hypothennuse AC.

*First,*

*First*, To delineate the  $\Delta$ , that the  $\angle$  required, viz. A, may be at the Centre.

Set the half Tangent of  $38^{\circ} 56'$  from A to B; and thro' the three Points Z B and N, draw an Ob-



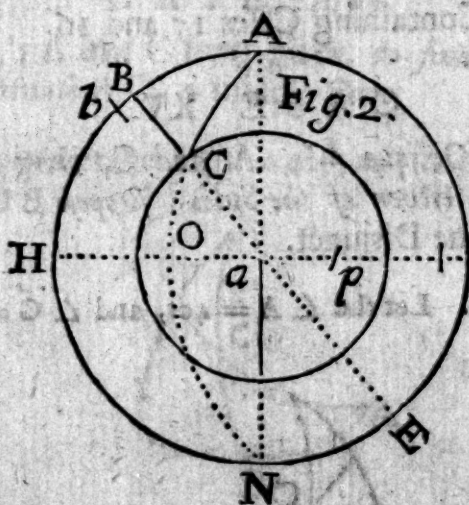
lique Circle, which will be at right  $\angle$ 's with AB. Then draw a Circle parallel to HO, at  $27^{\circ} 48'$  Distance from it, by *Prob. 8. Chap. 8.* and, lastly, thro' C, and the Centre A, draw the right Circle EF, and it's finish'd.

*Secondly*, To make the  $\Delta$ , that the  $\angle$  A may be at the Primitive Circle.

Draw a Circle parallel to the Primitive, at the Distance of BC, from it, (by *Prob. 8*) thus: With the half Tangent of  $62^{\circ} 12'$  (the Compl. of BC, and Distance of the Parallel from the Centre (a)), describe a Circle.

Make AB =  $38^{\circ} 56'$  by the Chords; and from B, and thro' the Centre (a) draw the Right Circle:

**BE.** Lastly, Thro' the three Points A, C, and N,  
draw the Oblique Circle, and it's done.



**The Parts required are measured thus:**

In Scheme the 1<sup>st</sup>, the Hypothenufe AC is mea-  
sur'd by half Tangents, and will be found equal to  
 $46^{\circ} 31'$ .

But in Scheme 2. it is measured by reducing it to the Primitive, as before directed; being the Arch  $A b = 46^{\circ} 31'$ . The Measure of the  $\angle A$ , in the first  $\Delta$ , is the Arch  $FO$ , and will be found equal to  $40^{\circ}$  upon the Chords. But in the second  $\Delta$ , the Quantity of the  $\angle A$  is  $HO$  measured upon the half Tangents from  $90^{\circ}$  downwards. See *Prob. 7.*

PROP.



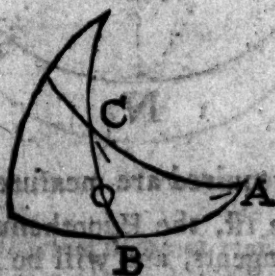
## P R O P. VI.

Containing Cases 15 and 16.

## C A S E X V.

The two Oblique  $\angle$ 's, A and C, being given,  
to find either of the Sides; suppose B C, an  
Extreme Disjunct.

*Example.* Let the  $\angle A = 40^\circ$ , and  $\angle C = 60^\circ$ .



Here the Complement of the  $\angle A$  is the *middle Part*, and the Complement of the  $\angle C$ , and the Side B C, are *Extremes Disjunct*; therefore the Equation is,

$$cs. \angle A 40^\circ + Rad. = s. \angle C 60^\circ + cs. BC 27^\circ 48'.$$

And the *Analogy*,

$$As, s. \angle C 60^\circ : cs. \angle A 40^\circ :: Rad : cs. BC = 27^\circ 48'.$$

Or *Instrumentally*.

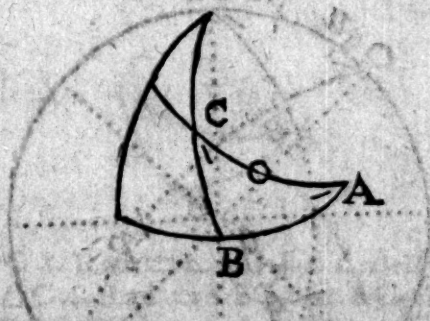
Extend from  $60^\circ$  to  $50^\circ$ , (the Complement of the  $\angle A$ ) the same will reach from  $90^\circ$  to  $62^\circ 12'$ , the Compl of BC =  $27^\circ 48'$ . Note,

Note, BC will be less than a Quadrant, when the  $\angle$  opposite thereunto is Acute; but greater, if Obtuse.

## C A S E XVI.

The  $\angle$ 's A and C being given, to find the Hypothenuse AC, a middle Part.

Example. Let the  $\angle A = 40^\circ$ , and  $\angle C = 60^\circ$ .



In this Case, the Complement of the Hypothenuse AC, being the Term required, is the *middle Part*, and the Complements of the  $\angle$ 's A and C are *Extremes Conjunct*: Ergo the Equation will be

$$\text{ct. } \angle A 40^\circ + \text{ct. } \angle C 60^\circ = \text{Rad.} + \text{cs. } AC 46^\circ 31'.$$

And the *Analogy*,

$$\text{As, Rad. : ct. } \angle A 40^\circ :: \text{ct. } \angle C 60^\circ : \text{cs. } AC = 46^\circ 31'.$$

Or *Instrumentally*.

Extend from the Tangent of  $50^\circ$  to  $30^\circ$ , that Extent will reach from  $90^\circ$  to  $43^\circ 29'$  in the Sines, the Complement of  $AC = 46^\circ 31'$ .

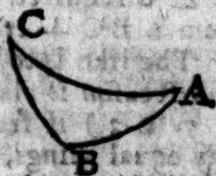
The Hypothenuse AC will be less than a Quadrant, if the  $\angle$ 's be of the same Affection; but greater, if they be of a different Affection. The



The Parts required are found thus:

The Measure of AC by the Chords is equal to  $46^{\circ} 31'$ . But the Measure of AB is the Arch  $A\pi = 38^{\circ} 56'$ , and the Measure of BC is the Arch  $C\pi = 27^{\circ} 48'$ ; both being found by reducing to the Primitive, as before directed.

Thus having resolv'd all the 16 Cases of Rectangled Spherical Triangles, here follows a brief View of the whole, comprized under six simple Equations.



1.  $S. BC + Rad. = s. AC + s. \angle A$ . Case 1, 4, & 9.
2.  $Cs. \angle C + Rad. = s. BC + cs. AC$ . Case 2, 5, & 12.
3.  $Cs. AC + Rad. = ct. \angle A + ct. \angle C$ . Case 3, & 16.
4.  $Cs. AC + Rad. = cs. BC + cs. AB$ . Case 6, & 14.
5.  $S. AB + Rad. = ct. \angle A + s. BC$ . Case 10, 7, & 13.
6.  $Cs. \angle A + Rad. = cs. BC + s. \angle C$ . Case 8, 11, & 15.

Now that all the rest of the 16 Cases are deducible from these six, as above, is, I think, of itself very apparent and clear. Nevertheless, for the Sake of some who may not of themselves so readily apprehend it, I shall instance in one or two of them, as follows.

In Case 1. where there are given AC and the  $\angle A$ , to find BC, which by the Equation is thus order'd:

$$s. BC + Rad. = s. AC + s. \angle A.$$

Now, let us suppose AC and BC were given, to find the  $\angle A$ . Agreeable to Case 4.

T

Then



Then seeing that the  $s. BC + Rad. = s. AC + s. \angle A$ , it will hence undoubtedly appear evident to a very mean Capacity, that if from the Sum of the Sine of  $AB$  and Radius, there be taken away the Sine of  $AC$ , the Remainder will be the Sine of  $A$ . And by the same way of reasoning, if we suppose  $BC$  and the  $\angle A$  given, to find  $AC$ , agreeable to Case 9. it's plain, that if from  $s. BC + Rad.$  there be taken the Sine of  $A$ , it will produce the Sine of  $AC$ .

Again; In the second Equation, let  $BC$  and  $AC$  be given, and the  $\angle C$  required (the same as in Case 5.) Then from  $s. BC + ct. AC - Rad.$  it gives  $C s. \angle C$ . The like is to be understood of all the rest; and the Reason is plain:

For (as by *Ax. 5. & 3.*) if from equal things, there be taken away equal things, the Remainders will be equal. So if  $6 + 10 = 9 + 7$ , then if from  $6 + 10 = 16$ , we take away 9, the Remainder is  $= 7$ ; or if we take away 7, the Remainder is 9, &c.

Note, In this place (as before) instead of subtracting a Sine or Tangent, you may add its Arithmetical Complement, which will produce the same Answer.

I judge it will not be unacceptable to a Learner, if I here shew him how to reduce these Equations into Analogies or Proportions, that so he may at any time work by that which he likes best.

The Rule for which is as follows, and is founded upon the 16th Prop. of Book 6. of Euclid's Elements.

Let the Number to be subtracted (which is always that which is signed with the Term required) be put in the first place, and the two Terms which are on the other Side of the Equation, the one in the second place, and the other in the third, it matters not which. For Trial

hereof, let us take the first Case above, where, as has been shown, there are suppos'd to be given,

1. AC and  $\angle A$ , to find BC.
2. AC and BC, to find the  $\angle A$ .
3. BC and  $\angle A$ , to find AC.

Which by the Equation is as follows:

1.  $s. \angle A + s. AC = (\text{Rad.}) + s. BC.$
2.  $s. BC + \text{Rad.} = (s. AC) + s. \angle A.$
3.  $s. BC + \text{Rad.} = (s. \angle A) + s. AC.$

And, by the Rule above, may be converted into Analogies, only by observing to set the Terms of the Equation, included within this Mark ( ) in the first place, which by the foregoing Directions are the Terms, in each Operation, to be subtracted from the Sum of the other two Terms on the contrary Side of the Equation; and so the Analogies will stand thus:

1.  $\text{Rad} : s. AC :: s. \angle A : s. BC.$
2.  $s. AC : \text{Rad} :: s. BC : s. \angle A.$
3.  $s. \angle A : s. BC :: \text{Rad} : s. AC.$

Or you may, if you please, change the Places of the two middle Terms, which will occasion no Alteration in the Answer; but that way is generally to be preferred, which disposes of the Logarithms for the greater Conveniency in working; and that is the chief Reason why I have sometimes disposed of the Terms of the Equations, contrary to the strict Order of the Rule in the foregoing Cases, as was noted before.

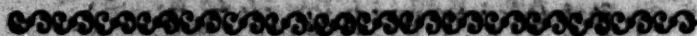
Here the Learner may observe, for his Advantage, that tho' in all the Proportions to the 16 Cases of Spherical Triangles, before set down and

explain'd, I have exactly pursu'd the Method prescribed by the universal Proposition; yet many of them may be so varied, as to bring the Radius into the first place of the Analogy, by taking notice only of what is mentioned before in Page 167. viz. *that Radius is a mean Proportional between the Tangent of an Arch and the Co-tangent of the same Arch.*

And therefore where it is at any time said, *As the Tangent is to Radius; to bring the Radius into the first place, we may say, As Radius is to the Co-tangent; which will produce the same thing.*

But the Reason of this Conversion (and some others, which I shall mention hereafter) will be most evident, when we come to treat of the Nature of Proportion, with its several Variations; and to set down and explain the chief Theorems on which they are founded.

Thus have I done with *Rectangled Sphericals*; it remains now, that I shew the Learner how to perform, after the same Method, the 12 Cases of *Oblique Sphericals*; which I shall endeavour, in the most brief and clear manner, in the next Section.



## S E C T. II.

### *Of Oblique Spherical Triangles.*

I HAVE before mentioned, that I did not intend to give any particular *Demonstration* of the *Universal Proposition*, but referred the Learner, for farther Satisfaction therein, to Mr. Norwood's *Doctrine of Triangles*\*; but it being thought needful, by some Persons of better Judgment, I shall pay

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\* Vid. Page 168.

them that Deference, as to insert it before I go any farther, tho' somewhat different from what that *Author* has done, and in such a manner as I apprehend most agreeable to a Beginner: In order whereunto, I shall premise the following *Lemma's*; which-being admitted, the Reason of this universal *Axiom* will plainly appear.

## L E M M A I.

*In all Right-angled Spherical Triangles, having the same Acute  $\angle$  at the Base, the Sines of the Bases are directly proportional to the Tangents of the Perpendiculars; & contra.*

i. e. As S, Base : Rad : : Tangent of the Perpendicular : Tangent  $\angle$  at the Base.

## L E M M A II.

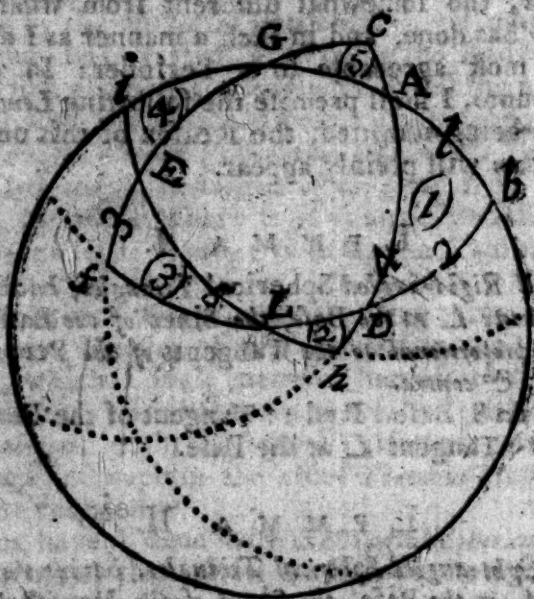
*In Right-angled Spherical Triangles, having the same Acute  $\angle$  at the Base, the Sines of the Hypothenuses and Perpendiculars are in a direct Proportion.*

i. e. As S, of the Hypothenuse : Rad : : S, of the Perpendicular : S,  $\angle$  at the Base.

## L E M M A III.

*If five great Circles of the Sphere be so described, that the first intersects the second, the second the third, the third the fourth, the fourth the fifth, and the fifth the first, at right  $\angle$ 's; the right  $\angle$ 's made by their Intersections, do all consist of the same circular Parts. This we shall make evident by the following Scheme.*





In which the first Arch intersects the second at (b), the second Arch the third at (f), the third Arch the fourth at (c), the fourth Arch the fifth at (h), and the fifth Arch the first at (i); which Intersections, b, f, c, h, i, are all at right  $\angle$ 's. Therefore the right  $\Delta^s$   $\Delta^s$  A b D, D b L, L f E, E i G, G c A, do all consist of the same circular Parts, as is manifest by what follows.

The five circular Parts in the Triangle A b D, are A b, b D, comp. b D A, comp. A D, comp. D A b.

In D b L, are comp. b L D, comp. L D, comp. L D b, D b, b L.

In L f E, are comp. E L f, L f, f E, comp. f E L, comp. E L.

In E i G, are i G, comp. i G E, comp. G E, comp. G E i, E i.

In G c A, are comp. G A, comp. A G c, G c, c A, comp. c A G.

Where.

Where it may be observ'd, that the Side  $AB$  in the first  $\Delta =$  Comp.  $BLD$ , in the second; Comp.  $ELf$ , in the third;  $iG$ , in the fourth; and Comp.  $GA$ , in the fifth; &c.

i. e. The Arch  $AB$ , in the first  $\Delta$ , is the same with the Comp.  $\angle BLD$ , in the second; Comp.  $\angle ELf$ , in the third; the Arch  $iG$ , in the fourth; and Comp. of the Arch  $GA$ , in the fifth; &c.

The same Uniformity of the circular Parts in Quadrantal Triangles might be also made apparent by the foregoing Scheme, but that I think is needless.

These Things premised, I shall again set down the *Universal Axiom*, as follows, and then explain it.

### R U L E I.

In any Right-angled Spherical Triangle, the Rectangle under the Radius, and the Sine of the middle Part, is equal to the Rectangle under the Tangents of the adjacent Extremes.

### R U L E II.

The Rectangle under the Radius, and the Sine of the middle Part, is equal to the Rectangle under the Cosine of the opposite Extremes.

Each of these Rules have three Cases: For the middle Part may be the Complement of one of the Oblique  $\angle$ 's, the Complement of the Hypotenuse, or one of the Sides.

The Learner may be admonished, before he proceed, to observe, that tho' I have here set down the *Universal Rule* in other Words than before in Page 166. yet if rightly considered, does amount to the same thing; for the Rectangle, or Product, of any two Quantities arising from the Multiplication thereof by natural Numbers, is all

all one with the Aggregate, or Sum of those Quantities found by Addition of the Artificial or Logarithmical Numbers: But we have before spoken of this, and therefore shall proceed in the same Method as at first.

CASE 1. Let the middle Part be a Side, as  $Ab$ , in the  $\triangle Abd$ , and  $Db$ , and Comp. of  $\angle A$  the Extremes adjacent. Then, by Lemma 1. As  $S$ ,  $Ab : Rad :: Tang. Db : Tang. \angle A$ . Therefore also alternately, As  $S$ ,  $Ab : Tang. Db :: Rad : Tang. \angle A$ .

But Radius is a mean Proportional between the Tangent of an Arch, and the Tangent Complement of that Arch: Wherefore, As  $Rad : T, \angle A :: Co-tangent of \angle A : Rad$ . Whence, As  $S$ ,  $Ab : T, Db :: Cr. \angle A : Rad$ . Therefore, (by 16. Encl. 6.)  $S, Ab \times Rad. = t. Db \times cr. \angle A$ . Which was to be proved.

CASE 2. Let the middle Part be the Comp. of the  $\angle L$ , in the  $\triangle bLD$ , and  $bL$ , and Complement of  $LD$ , the adjacent Extremes. Then, by Lemma 3. Compl.  $\angle bLD = Ab$ , and Compl. of  $LD$  is  $DB$ ; and  $bL$  is the Complement of the  $\angle DAb$ ; and we have already proved, in the last Case, that  $S, Ab + Rad. = T, Db + cr. \angle A$ . Therefore also,  $Cr. \angle bLD + Rad. = cr. LD + T.bL$ .

CASE 3. Let the Complement of the Hypotenuse  $AG$ , in the  $\triangle GcA$ , be the middle Part, and the Complement of the  $\angle's G$  and  $A$ , the adjacent Extremes. Then having proved, in the first Case, that  $S, Ab + Rad. = T, Db + Cr. \angle A$ ; but (per Lemma 3.) Comp.  $AG = Ab$ , Compl.  $\angle AGc = Db$ , and Compl.  $\angle cAG = Compl. \angle DAb$ : Ergo,  $Cr. AG + Rad. = Cr. \angle AGc + Cr. \angle cAG$ .

Therefore,

Therefore, in any right  $\angle^d$  Spherical Triangle, the Rectangle under the Radius, and the Sine of the middle Part, = the Rectangle under the Tangents of the adjacent Parts. Q. E. D.

To prove the second Part of the Rule.

CASE 1. Let the middle Part be a Side, as  $Db$ , in the Triangle  $AbD$ , and the Compl. of  $AD$ , and Compl.  $\angle A$ , the opposite Extremes. Then, per Lemma 2. As  $S, AD : Rad :: S, Db : S, \angle A$ . Wherefore,  $S, Db + Rad = S, AD + S, \angle A$ . Per 16. Euclid 6. as above.

CASE 2. Let the Complement of the Hypotenuse  $LD$ , in the  $\triangle DbL$ , be the middle Part, and the Sides  $Db$  and  $Lb$ , the opposite Extremes. Then, since we have before proved (per Lemma 3.) the Compl.  $LD = Db$ , and  $Db = \text{Compl. } AD$ ; also  $bL = \text{Compl. } \angle DAb$ ; and since, by the last Case,  $S, Db + Rad = S, AD + S, \angle A$ . Therefore,  $Cs, LD + Rad = Cs, Db + Cs, bL$ .

CASE 3. Let the middle Part be the Compl. of one of the Oblique  $\angle$ 's, as  $G$ , in the  $\triangle EiG$ ; and the Compl.  $\angle E$ , and  $Ei$ , the opposite Extremes. Then, by Lemma 2. As  $S, AD : Rad :: S, Db : S, \angle A$ . But  $Db = \text{Compl. } \angle iGE$ , and  $AD = \angle GEi$ , and the Compl.  $\angle A = Ei$ . Wherefore the  $Cs, \angle iGE + Rad = S, \angle GEi + Cs, Ei$ .

And therefore in any right  $\angle^d$  Spherical  $\triangle$ , the Rectangle under Radius, and Sine of middle Part = the Rectangle under the Co-sines of the opposite Parts. Q. E. D.

Thus I have endeavour'd to demonstrate this most excellent Proposition, in as plain and concise manner as I could think of, only for the Entertainment of the Curious; for I do not apprehend  
it



it necessary for every Beginner to trouble himself about it, tho' such as are ingenious may, and perhaps will, expect it should be given here.

I come now (as I intended to have done before) to give Directions for performing the several Cases of Oblique Spherical Triangles that are reducible to the *General Proposition*; and to lay down some things previous thereunto, to render them as easy and intelligible as possibly I can to common Apprehensions. And first of all, because in every of the Cases (which are Ten in Number) there is required the letting fall a Perpendicular, whereby the Oblique  $\angle^d$  Triangle is reduced into two right  $\angle^d$  ones, and so to be resolved by two Operations; it must therefore be of Service to the Learner, if we here exhibit the general Method of letting fall the Perpendicular, which may be comprized in the following

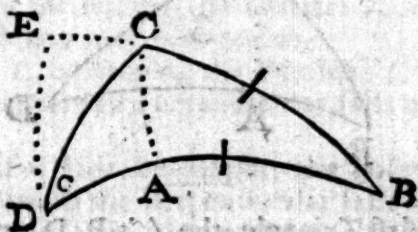
### R U L E.

*Let the Perpendicular fall from the End of a given Side; and let it subtend an adjacent  $\angle$  given. And farther, when that is not sufficient, (as when the three Things given are Conterminal, or adjoining, and an  $\angle$  is required) let it fall opposite to the  $\angle$  required, or next to the Side required, if it be a Side that is sought.*

To illustrate both the Parts of this Rule; in the Triangle BCD, let BD, BC and  $\angle$  B, be given; and the  $\angle$  D required.

Here, if we regard only the former Part of the Rule, it may be doubtful whether the Perpendicular falls from C or D; for either way it will fall from the End of a given Side, and will subtend the given  $\angle$  B; wherefore to determine in such Cases, the latter Part of the Rule must be observed. Whence we shall perceive the Perpendicular to fall, in this Instance, from C, and not D;

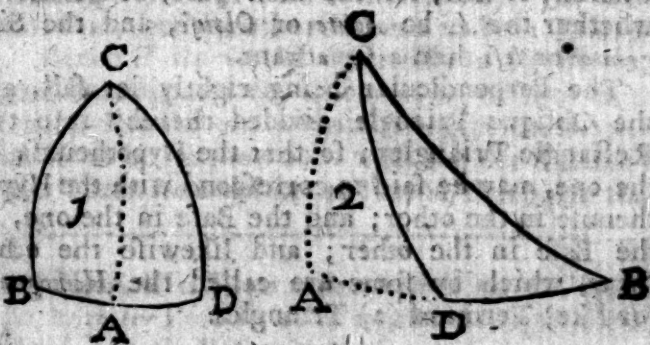
D; for if otherwise, it cannot subtend the required  $\angle D$ , as that Part of the Rule directs.

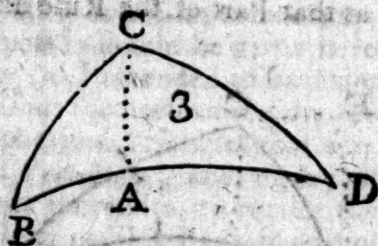


But now it may hence be observed, that the Perpendicular falls sometimes within, and sometimes without the Triangle; and that you may at any time know which, take this *General Rule*.

*In any Spherical Triangle, if the  $\angle$ 's at the Base (or Side upon which the Perpendicular falls) be of the same Affection, i. e. of one Specie, then the Perpendicular falls within the Triangle; and if they be of a different Affection or Specie, the Perpendicular falls without the Triangle.*

See the Examples following, which will explain this better than many Words.





In the first Triangle, the  $\angle$ 's B, D, are both *Obtuse*: In the second Triangle, the  $\angle$ 's B, D, are one *Acute*, and one *Obtuse*; and in the third Triangle, the  $\angle$ 's B, D, are both *Acute*. In each of which Triangles, the Perpendicular A C falls accordingly.

Note, It may sometimes appear doubtful, whether the  $\angle$  nearest to a Right  $\angle$ , and its opposite Side be of the same, or of a different Affection, unless it be discovered by working, or that it be given by Supposition.

But this Doubt may be removed by *Defin. 14. Chap. 8. §. 1. of Spherical Triangles*, and the Converse.

Or by an exact Delineation of the Triangle; whereby it will, for the most part, be perceived whether the  $\angle$  be *Acute* or *Obtuse*, and the Side greater or less than a Quadrant.

The Perpendicular being rightly let fall, and the Oblique Triangle divided thereby into two Rectangle Triangles; so that the Hypothenuse in the one, may be said to correspond with the Hypothenuse in the other; and the Base in the one, to the Base in the other; and likewise the other Parts, which by some are called the *Homogenial* (or like) Terms of (2) Triangles.

Then

Then in the first of those Rectangle  $\Delta$ 's, viz. That wherein three Things (with the right  $\angle$ ) are given, will the first Operation fall; and that will be either for finding the Vertical  $\angle$ , or the Base. In which you may take notice,

If an  $\angle$  be sought, the Vertical  $\angle$  must first be found, unless the Perpendicular falls upon a known Side.

But if a Side be sought, the Base must be first found, unless the Perpendicular falls from a known Angle; for then the contrary must be observed every way.

The second Operation lies in comparing the *Homogenial* (or like) Terms of both Triangles between themselves, i. e. two given Things in one Triangle, to two correspondent Things in the other Triangle \*. Which two in each, with the Perpendicular, make three Things in each Triangle either *contiguous* (that is, lying together) or opposite; of which three the Perpendicular is always one (and the same) of the *Extremes* in both Triangles; as will appear very plain by the Triangles themselves, and shall be farther confirmed when we come to the Examples or Cases. In the mean while let it be observed, that tho' the Perpendicular be always one of the *Extremes* in either Triangle (as was before mention'd); yet we use not that, but the other *Extreme* in both Triangles.

Hence if we would determine a Proportion of two Homogenial (or like) Terms of both  $\Delta$ 's between themselves; we must imagine the Perpendicular to be compared both with the Term given, and the Term demanded in the second Triangle, that so

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\* Note, The Terms to be compared are the Hypothenu-ses with the Vertical  $\angle$ 's, or with the Bases; and also the  $\angle$ 's at the Base with the same.



it may be perceived which of those Terms may be made the *middle Part*, and which is the one *Extreme*, and of what kind it is; whereupon *Sines* or *Tangents* may be fitted to them, according to the *Universal Proposition*; to which do answer in all things, the *like Parts* of the first Triangle: Then

the Perpendicular being rejected, together with the Radius; we compare the *middle Part* and one of the *Extremes* in the first Triangle, with the *middle Part* and one of the *Extremes* of the second. Or, to speak more properly, according to the Words of the *Universal Proposition*; the *middle Part* in the first Triangle, with the *Extreme* in the second; is equal to the *middle Part* in the second, with the *Extreme* in the first.

Before I come to the Calculation of Oblique Spherical Triangles, I shall lay down six *Problems*, which comprehend the whole Variety of the Cases thereto belonging.

### PROBLEM I.

Two Sides and an Angle comprehended between them given, to find,

1. Either of the other  $\angle$ 's.
2. The third Side, (or Side opposite to the given Angle.)

### PROBLEM II.

Two Angles and a Side included between them given, to find,

1. Either of the other Sides.
2. The third Angle.

PROB-

**PROBLEM III.**

Two Angles and a Side opposite to one of them given, to find

1. The interjacent Side.
2. The third Angle.
3. The Side opposite to the other  $\angle$ .

**PROBLEM IV.**

Two Sides and an Angle opposite to one of them given, to find

1. The third Side.
2. The  $\angle$  comprehended between them.
3. The  $\angle$  opposite to the other Side.

**PROBLEM V.**

The three Sides given, to find

Either of the  $\angle$ 's.

**PROBLEM VI.**

The three Angles given, to find

Either of the Sides.

Thus having shew you all the Varieties that can possibly happen in the Resolution of Oblique-angl'd Triangles, under six general Propositions; I shall, in the next place, set down the Examples to all the twelve Cases; and such Instructions to every Case, as will be sufficient to inform the meanest Understanding of the Manner of their Operation.

PROBLEM I.

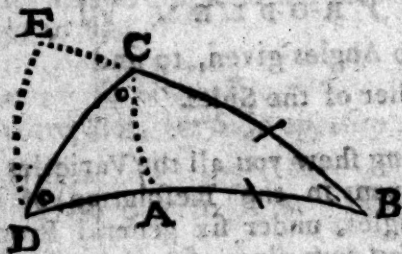
*Containing Cases first and second.*

Two Sides (BC) and (BD), and an  $\angle$  (B) comprehended, being given, to find,

CASE I. Either of the other  $\angle$ 's (D) or (BCD).

*Example.* Let  $BC = 30^\circ 00'$ ,  $BD = 42^\circ 09'$ ,  $\angle B = 36^\circ 08'$ , and  $\angle D$  be required.

Having let fall the Perpendicular (AC) by the latter Part of the Rule in Page 214, the first Operation is in the Triangle BAC; in which there is given the Hypotenuse BC and  $\angle$  B, to find the Segment of the Base BA. And therefore the Complement of the  $\angle$  B is the middle Part and Compl. of BC, and BA *Extremes Conjunct*; wherefore the Operation will be the same as in Case 2, of Right-angle Sphericks.



$$\text{cs. } \angle B + \text{Rad.} = \text{cs. } BC + \text{t. } BA.$$

That is,

$$\text{As cs. } BC 30^\circ : \text{Rad.} :: (\text{cs. } \angle B 36^\circ 08') : \text{t. } BA = 25^\circ 00'.$$

Note,

Note, For the more ready assisting of the Eye, I shall include the middle Part within this Mark ( ) in all the Proportions that follow, except such as are expressed in Words at length.

Subtract  $AB\ 25^{\circ}\ 00'$ , from  $BD\ 42^{\circ}\ 09'$ , there remains the Sum of  $AD\ 17^{\circ}\ 09'$ .

Which being found, the next thing must be to compare the  $\angle B$ , the Segment  $BA$ , and the Perpendicular  $AC$ ; in which we shall find  $BA$  the middle Part, and the Complement of  $B$ ; and Perpendicular, *Extremes Conjunct*. And so likewise in the Triangle  $CAD$ , the Segment  $AD$  (given by the last Operation) is the middle Part, if compar'd with the Perpendicular  $AC$ , and  $D$  the  $\angle$  required; and therefore rejecting the Perpendicular and Radius, it will be (by the Rule aforementioned in fol. 218.)

$$S, AB + tc. \angle D = tc. \angle B + s. AD.$$

Or Analogically.

As  $S, AB\ 25^{\circ}\ 00'$  the middle Part in the first  $\Delta$ ,

To  $tc. \angle B\ 36^{\circ}\ 08'$  the Extreme in the first;

So  $S, AD\ 17^{\circ}\ 09'$  the middle Part in the second,

To  $tc. \angle D\ 46^{\circ}\ 18'$  the Extreme in the second.

From the same Data as before.

Example 2. To find the  $\angle BCD$ .

Having let fall the Perpendicular  $DE$ , according as the Rule directs; then first find the Segment  $CE$ , by Case the second of Rectangled Sphericks,

$$cs. \angle B + Rad. = ct. EC + s. BE.$$

Or Analogically.

As  $ct. BC\ 30^{\circ}\ 00' : Rad :: cs. \angle B\ 36^{\circ}\ 08' : s. BE\ 36^{\circ}\ 10'$ .

Subtract  $BC$  from  $BE$ , there remains  $CE = 6^{\circ}\ 10'$ . U 3 Then



Then, *Secondly*, in the  $\triangle BDE$ , if we compare the  $\angle B$ , and the Base  $BE$ , with the Perpendicular  $DE$ ;  $BE$  will be the *middle Part* and Complement of the  $\angle B$ , and the Perpendicular  $DE$  *Extremes Conjunct*: and in the  $\triangle CDE$ ;  $CE$ , the given Term, will be the *middle Part*, if compared with the Perpendicular  $DE$ , and the  $\angle DCE$  the Term sought. Hence the Operation will be

$$S, BE + 2. \angle DCE = 2. CE + ct. \angle B.$$

And *Analogically*.

As 5,  $BE\ 36^\circ 10'$  the *middle Part* in the first  $\triangle$ ,

To  $ct. \angle B\ 36^\circ 08'$  an *Extreme* in the first;

So 2.  $CE\ 6^\circ 10'$  the *middle Part* in the second,

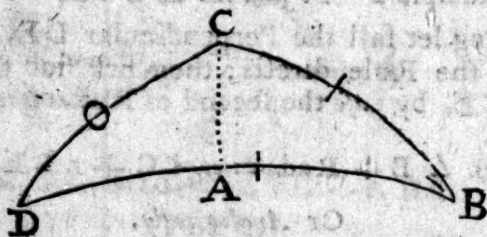
To 2.  $\angle DCE\ 76^\circ 00'$  an *Extreme* in the second.

Subtract  $\angle DCE$  from  $180^\circ$ , and it gives the  $\angle BCD = 104^\circ 00'$ .

## CASE II.

The same things as in the last Case given, to find the third Side  $CD$ .

Example. In the  $\triangle ABC$ , let  $\left\{ \begin{array}{l} BC \\ BD \\ \text{and } \angle B \end{array} \right\} = \left\{ \begin{array}{l} 30^\circ 00' \\ 42^\circ 09' \\ 36^\circ 08' \end{array} \right\}$  as before.



The Perpendicular being let fall from  $C$  upon the Base  $BD$ ; then to find the Segment thereof  $BA$ ;

BA; the Equation, by the second Case of Rect-  
angl'd Sphericks, will be

$$\text{cs. } \angle B + \text{Rad} = \text{cs. } BC + \text{t. } AB.$$

And the *Analogy*.

As  $\text{cs. } BC \ 30^{\circ} \ 00' : \text{Rad} :: (\text{cs. } \angle B \ 36^{\circ} \ 08') : \text{t. } AB = 25^{\circ} \ 00'.$

From BD subtra $\hat{c}$ t AB, the Remainder is AD  
 $= 17^{\circ} \ 09'.$

Secondly, Compare AB, BC, and the Perpendi-  
cular AC; and you'll find the Complement of BC,  
*middle Part*; and AB, AC, *Extremes Disjunct*: and  
in the  $\triangle DAC$ ; DC, the Term required, is the  
*middle Part*, and the given Term AD, and the Per-  
pendicular AC, the *Extremes*: Wherefore it will  
be by the Equation,

$$\text{cs. } BC + \text{cs. } AD = \text{cs. } AB + \text{cs. } CD.$$

And by *Analogy*.

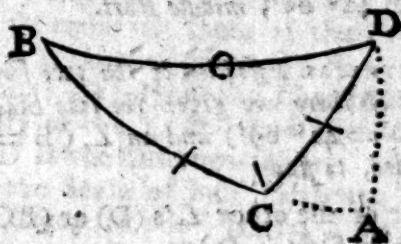
As  $\text{cs. } AB \ 25^{\circ} \ 00'$  an *Extreme* in the first  $\triangle$ ,

To  $\text{cs. } BC \ 30^{\circ} \ 00'$  *middle Part*;

So  $\text{cs. } AD \ 17^{\circ} \ 09'$  an *Extreme* in the second,

To  $\text{cs. } CD \ 24^{\circ} \ 04'$  *middle Part*.

*Example 2.* Let us suppose to be given  $BC = 30^{\circ} \ 00'$ ,  $CD = 24^{\circ} \ 04'$ , and the  $\angle C$  contained  
between, to find the Side BD.



In the  $\Delta$  CAD, the Hypotenuse CD, and  $\angle$  ACD, (being the Complement of  $\angle$  DCB) are given, to find the Base AC, by the second Case of Rectangle Sphericks, as follows.

Note, Here the Perpendicular may fall from D, or B, upon the Side BC, or DC extended; for either way it answers to the Rule: But we have here let it fall from D, upon the Side BC being continued; wherefore the Equation is,

$$cs. \angle ACD + Rad = ct. DC + t. AC.$$

And by Analogy.

As  $ct. CD 24^{\circ} 04'$  :  $Rad :: (cs. \angle ACD 76^{\circ} 00')$   
:  $t. AC = 6^{\circ} 10'$ .

To BC add AC, their Sum is the whole Base.  
 $AB = 36^{\circ} 10'$ .

Then, Secondly, by comparing two like things of one  $\Delta$  with two like things of the other  $\Delta$ , as is done before, the Terms of the Equation will stand thus:

$$cs. DC + cs. AB = cs. AC + cs. DB.$$

And by Analogy.

As  $cs. AC 6^{\circ} 10'$ , an Extreme in the first  $\Delta$ ,

To  $cs. DC 24^{\circ} 04'$ , middle Part;

So  $cs. AB 36^{\circ} 10'$ , an Extreme in the second,

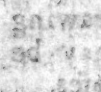
To  $cs. DB 42^{\circ} 09'$ , middle Part.

The GEOMETRICAL Effect of the two foregoing Cases, in which there are given the two Sides ( $BC = 30^{\circ}$ ) ( $BD = 42^{\circ} 09'$ ) and an  $\angle$  ( $B = 36^{\circ} 08'$ ) comprehended, to find

Either of the other  $\angle$ 's (D) or (BCD.)

The third Side (CD.)

•



Th

is the

*Lastly,*



*Lastly*, To find the Measure of the Side DC; Lay a Ruler from D to (b), it will cut the Primitive in (o); measure C (o) upon the Chords, and you'll find it  $24^{\circ} 04'$ , the Side DC.

*Note*, There's no Necessity for drawing the Parallel EDF, to set off the Number of Degrees required upon the Oblique Circle BDN, since it may as well be done otherwise; for having found the Pole (a) of the given Oblique Circle, set the Number of Degrees on the Primitive; and reduce it from thence to the Oblique Circle by the help of its Pole. See Prob. 5. Case 3.

Thus, if we had taken  $42^{\circ} 04'$  (the Side BD) from the Chords, and set from B to E; a Ruler laid over C and (a) would have cut the Oblique Circle BDN, in D; and so BD shou'd have been  $= 42^{\circ} 04'$ , as was obtained before, by drawing the Parallel EDF. Which Admonition may be observed by a Learner in any of the like Cases.

## P R O B L E M II.

*Containing the third and fourth Case.*

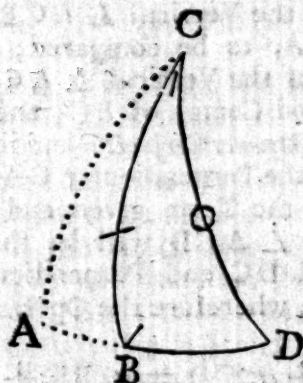
Two  $\angle$ 's (B) and (C), and a Side (BC) included between them, given,

CASE 3. To find either of the other Sides CD and BD.

*Example.*

Let  $\angle DCB = 36^{\circ} 08'$ .  $\angle CBD = 104^{\circ} 00'$ , and  $CB = 30^{\circ} 00'$ .

The



The Perpendicular being let fall, according to the Rule, it will, in this Case, fall from the  $\angle$  C, and the Extremity of the known Side BC, and will subtend the adjacent given  $\angle$  CBA; and because the  $\angle$ 's D and B at the Base are of a different Affection, it falls without the Triangle, and reduces it into two Right-angled Triangles, viz. CAB and CAD; in the first of which there is given the  $\angle$  B, and Side BC, to find the Vertical  $\angle$  C, by the third Case of Rectangle Sphericks, as underneath.

Note, The  $\angle$  CBA is known, being the Complement of the given  $\angle$  CBD to  $180^\circ$ , per Def. 8. fol. 145.

$$cs. BC + Rad = ct. \angle B + ct. \angle BCA.$$

And by Analogy.

$$As ct. \angle B 76^\circ 00' : (cs. BC 30^\circ 00') :: Rad : ct. \angle BCA = 16^\circ 04'.$$

Add  $\angle$  BCA to  $\angle$  BCD, and you have the  $\angle$  ACD.

In

In the next place, to find DC, we must imagine the Side BC, the Vertical  $\angle ACB$ , and the Perpendicular CA, to be compared; and then the Complement of the Vertical  $\angle ACB$  will be the *middle Part*, and Compl. of BC; and the Perpendicular CA, *Extremes Conjunct*: and in the  $\triangle ADC$ , if we suppose the Perpendicular CA, the  $\angle ACD$  and Side DC, the Term given and sought, to be compared, the  $\angle ACD$  will be the *middle Part*, and the Compl. DC and Perpendicular AC, *Extremes Conjunct*; wherefore the Operation by the Equation is,

$$cs. \angle ACB + ct. CD = ct. BC + cs. \angle ACD.$$

And by *Analogy*.

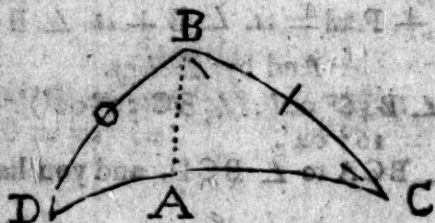
As *cs.*  $\angle ACB$   $16^{\circ} 04'$ , the *middle Part* in the 1st  $\triangle$ ,  
To *ct.* BC  $30^{\circ} 00'$ , an *Extreme* in the first;  
So *cs.*  $\angle ACD$   $52^{\circ} 12'$ , the *middle Part* in the 2d  $\triangle$ ,  
To *ct.* DC  $42^{\circ} 09'$ , an *Extreme* in the second.

*Example 2.*

Again, from the same things given, to find the Side BD.

Here the Perpendicular must fall from B upon the Side CD by the foregoing Rule; and because the  $\angle$ 's at the Base are both of one kind, therefore it falls within the Triangle; and the first Operation will be to find the Vertical  $\angle CBA$  by the third *Case* of Rectangle Sphericks, thus:

$$cs. BC + Rad = ct. \angle C + ct. \angle CBA.$$



And

And the *Analogy*.

As *ct.*  $\angle C$   $36^{\circ} 08'$  : (*ct.*  $BC$   $30^{\circ} 00'$ ) : : *Rad* :  
*ct.*  $\angle CBA = 57^{\circ} 42'$ .

Subtra& the  $\angle CBA$  from the whole  $\angle CBD$ ,  
 there remains the  $\angle ABD = 46^{\circ} 18'$ .

2dly, Compare  $BC$ , the Vertical  $\angle CBA$ , and the Perpendicular  $AB$ ; and the Complement of the  $\angle CBA$  is the *middle Part*, and  $BC$  and  $AB$  *Extremes Conjunct*; and the other Vertical  $\angle ABD$ , in the second  $\Delta$ , is the *middle Part*, if compared with the Perpendicular  $BA$  and  $BD$ , the Term sought: and the Equation in the Homogeneous Terms of both Triangles will be thus;

$$ct. \angle CBA + ct. BD = ct. BC + ct. \angle ABD.$$

And the *Analogy*.

As *ct.*  $\angle CBA$   $57^{\circ} 42'$ , a *middle Part* in the first  $\Delta$ ,

To *ct.*  $BC$   $30^{\circ} 00'$ , an *Extreme* in the first;

So *ct.*  $\angle ABD$   $46^{\circ} 18'$ , *middle Part* in the second  $\Delta$ ,

To *ct.*  $BD$   $24^{\circ} 04'$ , an *Extreme* in the second.

#### C A S E IV.

From the same Data as in last Case, viz. two  
 $\angle$ 's ( $B$ ) and ( $C$ ), with a Side included be-  
 tween them ( $BC$ ), to find the other Angle  $D$ .

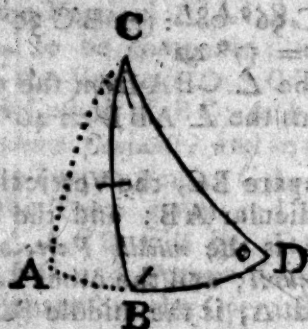
*Example 1.*

Suppose  $\angle DCB = 36^{\circ} 08'$ ,  $\angle CBD = 104^{\circ} 00'$ ,  $BC = 30^{\circ} 00'$ , as before in the last Case.

X

Here





Here the Perpendicular must fall, as in the last Case, by the Rules aforementioned; and then in the Triangle ABC, there are given the  $\angle ABC$  and BC, to find the Vertical  $\angle ACB$  by the third Case of Rectangle Sphericks, as before,

$$cs. BC \div Rad. = cs. \angle ABC \div cs. ACB.$$

Or thus:

$$As\ cs.\ \angle ABC\ 76^{\circ}00' : (cs.\ BC\ 30^{\circ}00') :: Rad. : cs.\ \angle ACB = 16^{\circ}04'.$$

The Sum of the  $\angle$ 's ACB and BCD is the whole  $\angle ACD = 52^{\circ}12'$ .

Then in the  $\triangle ABC$ , if the Perpendicular AC and the two oblique  $\angle$ 's B and C be compared, the Compl. of the  $\angle ABC$  is the *middle Part*, and the Compl. of the  $\angle ACB$  and AC, *Extremes Disjunct*; and in the  $\triangle ACD$ , the  $\angle D$  is the *middle Part*, and the Perpendicular AC and Compl.  $\angle ACD$ , the *Extremes*; wherefore rejecting the Perpendicular AC, as was directed before, and must be all along remember'd, the Equation will be as follows:

$$s.\ \angle ACB \div cs.\ \angle D = cs.\ \angle ABC \div s.\ \angle ACD.$$

And

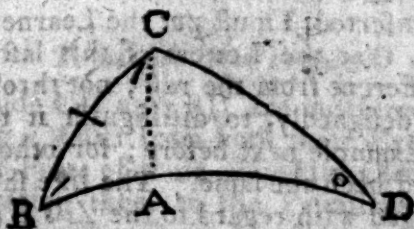
And the *Analogy*.

As *s.*  $\angle ACB$   $16^{\circ} 04'$ , an *Extreme* in the first  $\Delta$ ,  
 To *cs.*  $\angle ABC$   $76^{\circ} 00'$ , *middle Part* ;  
 So *s.*  $\angle ACD$   $52^{\circ} 12'$ , an *Extreme* in the second  $\Delta$ ,  
 To *cs.*  $\angle D$   $46^{\circ} 18'$ , *middle Part*.

*Example 2.*

Again ; Let us suppose the  $\angle C$  and  $B$ , and  $BC$ , the included Side, to be given, and the Angle  $D$  required, as in the following Scheme.

And then the Perpendicular will fall within the  $\Delta$  from  $C$ , upon the Base  $BD$  ; because the  $\angle$ 's at the Base are alike, *viz.* both Acute.



And the first Operation will be to find the  $\angle$  at the Perpendicular  $C$ , in the  $\Delta BCA$ , by the third *Case* of *Rectangle Sphericks*, thus :

Let  $\angle B = 46^{\circ} 18'$ ,  $\angle C = 104^{\circ} 00'$ , and  $BC = 24^{\circ} 04'$ .

Then *cs.*  $BC + \text{Rad.} = \text{cs. } \angle B + \text{cs. } \angle C$ .

And therefore by *Analogy*.

As *cs.*  $\angle B$   $16^{\circ} 18'$  : (*cs.*  $BC$   $24^{\circ} 04'$ ) :: *Rad.* : *cs.*  $\angle C = 46^{\circ} 18'$ .

Which subtracted from  $104^{\circ}$  (the Quantity of the whole  $\angle$  at  $C$ ) leaves the Vertical  $\angle ACD = 57^{\circ} 42'$ . X 2. Having

Having found the  $\angle C$  in the  $\triangle ACD$ ; if we next consider the Homogeneous Terms in both  $\triangle$ 's, viz. the  $\angle$ 's  $ABC$ ,  $BCA$ , in the first  $\triangle$ ; and the  $\angle$ 's  $CDA$ ,  $ACD$ , in the second; and compare them with the Perpendicular, as before, it will be, by the Equation,

$$s, \angle BCA + cs. \angle D = s, \angle ACD + cs. \angle ABC.$$

And by *Analogy*,

As  $s, \angle BCA 46^\circ 18'$ , an *Ext. Disj.* in the first  $\triangle$ ,

To  $cs. \angle ABC 46^\circ 18'$ , *middle Part*;

So  $s, \angle ACD 51^\circ 42'$ , an *Extremo* in the second  $\triangle$ ,

To  $cs. \angle D 36^\circ 08'$ , *middle Part*.

To prevent any Blunder being made, in comparing these  $\angle$ 's with the Tables of Logarithms hereafter inserted, I must put the Learner in mind to observe, that the Letters of this last Triangle are set different from the rest; not thro' any Mistake, but designedly, to distinguish it the better from the Example next before; for otherwise the Answer had been the same. The like should have been observed with regard to the  $\angle$ 's  $B$  and  $C$ , in the Example before this, and in both the Examples to *Case 3.* but was there forgot.

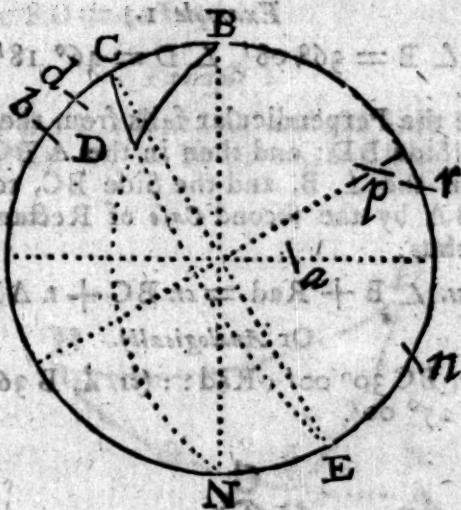
To delineate the  $\triangle$ , and solve these two Cases Geometrically; in each of which there are given,

The two  $\angle$ 's  $\left\{ \begin{array}{l} B = 46^\circ 18' \\ C = 104^\circ 00' \end{array} \right\}$  and a Side included  $BC 24^\circ 04'$ ,

To find the other Sides  $CD$ ,  $BD$ , and the third Angle  $D$ .

Make the  $\angle B = 36^\circ 08'$ , and the  $\angle C = 104^\circ$ , by drawing the Oblique Circles  $BDN$  and  $CDE$ , (by *Prob. 3. Chap. 8.*) the Side  $BC$  being first set off from the Chords.

To



*To measure what's required.*

The Side CD and BD are measured as before, by laying a Ruler from both their Extremities at D, to the Poles of their respective Circles *a* and *p*, and reducing them to the Primitive at *b* and *d*; so is *Bd* ( $= 42^{\circ} 09'$ ) the Measure of BD, and *Cb* ( $= 24^{\circ} 04'$ ) the Measure of CD. After the same manner is found the Arch ( $\vee$ ) (*r*), the Quantity of the  $\angle D = 46^{\circ} 18'$ .

### PROBLEM III.

*Containing the fifth, sixth, and seventh Cases.*

Two  $\angle$ 's B and D, and an opposite Side to one of them (BC) given,

CASE 5. To find the interjacent Side BD.

X 3

*Example*



*Example 1.*

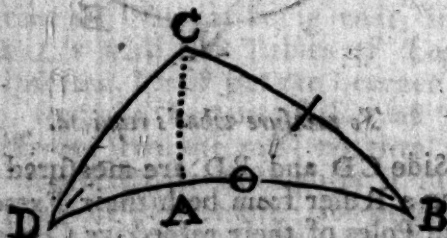
Let  $\angle B = 36^\circ 08'$ ,  $\angle D = 46^\circ 18'$ , and  $BC = 30^\circ$ .

Here the Perpendicular falls from the  $\angle C$  upon the Side  $BD$ ; and then in the  $\Delta BCA$ , there is given the  $\angle B$ , and the Side  $BC$ , to find the Base  $BA$  by the second *Case* of *Rectangle Sphericks*, thus:

$$\text{cs. } \angle B + \text{Rad} = \text{ct. } BC + \text{t. } AB.$$

Or *Analogically*.

$$\text{As ct. } BC 30^\circ 00' : \text{Rad} :: (\text{cs. } \angle B 36^\circ 08') : \text{t. } AB = 25^\circ 00'.$$



Then, *Secondly*, if we compare the  $\angle B$ , the Base  $BA$ , and the Perpendicular  $AC$ , the Base  $AB$  will be the *middle Part*, and the Complement of the  $\angle B$  and  $AC$ , *Extremes Conjoint*; and in the  $\Delta CAD$  of the three Terms to be compared, *viz.* the Base  $AD$ , the  $\angle D$ , and the Perpendicular  $AC$ ;  $AD$  is the *middle Part* and Compl. of the  $\angle D$ , and the Perpendicular *Extremes*: Therefore the *Equation* is,

$$\text{cs. } \angle B + \text{t. } AD = \text{s. } AB + \text{ct. } \angle D.$$

And the *Analogy*,

As  $\text{ct. } \angle B 36^\circ 08'$ , an *Extreme* in the first  $\Delta$ ,

To  $\text{s. } AB 25^\circ 00'$ , *middle Part*;

So  $\text{ct. } \angle D 46^\circ 18'$ , an *Extreme* in the second  $\Delta$ ,

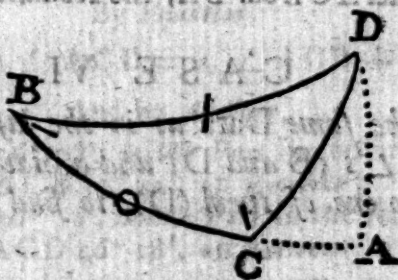
To  $\text{t. } AD 17^\circ 09'$ , *middle Part*.

Add

Add AB and AD together, and you have the whole Side BD =  $42^{\circ} 09'$ .

*Example 2.*

Suppose the  $\angle$ 's B and C, and the Side BD, in the following  $\Delta$ , to be given, and the Side BC required.



In this Case the Perpendicular falls without the  $\Delta$ , from the  $\angle$  D upon the Base BC (extended) because the  $\angle$ 's C and D at the Base are of a different Affection; and then in the  $\Delta$  ABD, there is given BD =  $42^{\circ} 09'$ , and the  $\angle$  B =  $36^{\circ} 08'$ , to find the Base AB by the second Case of Right-angle Sphericks, thus:

$$\text{cs. } \angle B + \text{Rad.} = \text{cs. BD} + \text{t. BA.}$$

Or Proportionally,  $\text{cs. } \angle B + \text{Rad.} : \text{cs. BD} :: \text{t. BA} : \text{t. BA}$

$$\text{As cs. BD } 42^{\circ} 09' : \text{Rad.} :: (\text{cs. } \angle B \ 36^{\circ} 08') = \text{t. BA} = 36^{\circ} 10'.$$

Again, Secondly, By comparing the  $\angle$  B, the Base BA, and Perpendicular AD, we shall find BA the *middle Part*, and Complement of the  $\angle$  B and AD *Extremes Conjoint*; and in the other  $\Delta$ , AC is the *middle Part*, and the Compl. of the  $\angle$  ACB, and Perpendicular AD, *Extremes*; and therefore the Equation is,

ed:  $\angle B + \angle A C = 180^\circ$ ,  $AB + \angle DCA$ , (the Complement of the  $\angle BCD = 104^\circ$ .)

And the *Analogy*,

As  $\angle B 36^\circ 08'$ , an *Extreme* in the first  $\Delta$ ,

To  $s. BA 36^\circ 10'$ , *middle Part*;

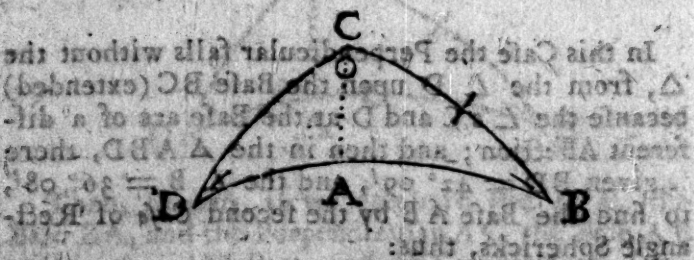
So  $\angle DCA 76^\circ 00'$ , an *Extreme* in the 2d  $\Delta$ ,

To  $s. AC 6^\circ 10'$ , *middle Part*.

Subtract  $AC$  from  $BA$ , the Remainder is  $BC = 30^\circ$ .

## CASE VI.

From the same Data as in the last Case, viz. two  $\angle$ 's ( $B$  and  $D$ ) and a Side ( $BC$ ) opposite to one of them ( $D$ ), to find the third  $\angle$  ( $C$ ).



*Example 1.*  
Let  $\angle B = 36^\circ 08'$ ,  $\angle D = 46^\circ 18'$ ; and  $BC = 30^\circ$ .

Here the Perpendicular falls within the  $\Delta$ ; and the first Operation will be to find the Vertical  $\angle C$  in the  $\Delta ACB$ , by the third Case of Rectangle Sphericks.

$$s. BC + \text{Rad} = \angle B + \angle ACB.$$

Or,

$$\text{As } \angle B 36^\circ 08' : \text{Rad } 90^\circ :: (s. BC 30^\circ) : \angle ACB = 37^\circ 42'.$$

Then

Then, *Secondly*, if we compare the  $\angle$ 's B and A C B, and the Perpendicular A C, in the  $\triangle A B C$ ; the Compl. of the  $\angle$  B will be the *middle Part*, and A C and the Compl. of the  $\angle$  A C B, *Extremes Disjunct*: and in the other  $\triangle$  it will be the  $\angle$  D *middle Part*, and A C and Compl. of the  $\angle$  C, *Extremes*; wherefore the second Operation will be thus:

By Equation,

$$cs. \angle B + s. \angle A C D = s. \angle A C B + cs. \angle D.$$

And by *Analogy*,

As *cs.*  $\angle B$   $36^{\circ} 08'$ , *middle Part* in the first  $\triangle$ ,

To *s.*  $\angle A C B$   $57^{\circ} 42'$ , an *Extreme*;

So *cs.*  $\angle D$   $46^{\circ} 18'$ , *middle Part* in the second  $\triangle$ ,

To *s.*  $\angle A C D$   $46^{\circ} 18'$ , an *Extreme*.

Add the  $\angle$  A C B to the  $\angle$  A C D, their Sum is the whole  $\angle$  at C  $= 104^{\circ}$ .

### Example 2.

Suppose the  $\angle$ 's B and C, and the Side B D, given to find the  $\angle$  D.

Let  $\angle B$   $36^{\circ} 08'$ ,  $\angle C$   $104^{\circ}$ , and B D  $42^{\circ} 09'$ .

In the  $\triangle A B D$ , there is given the  $\angle$  B, and Hypothenuse B D; to find (by the third *Case* of *Rectangle Sphericks*) the  $\angle$  A D B; in this manner,

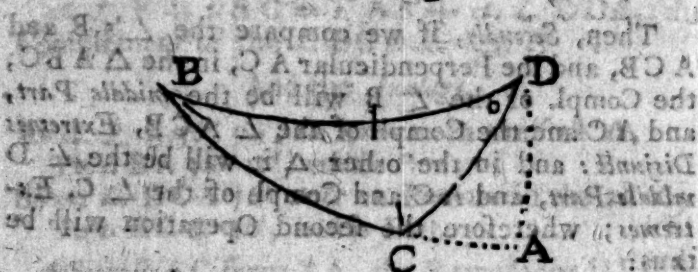
$$cs. B D + Rad. = cs. \angle B + cs. \angle A D B.$$

Or thus:

$$As \text{cs. } \angle B \ 36^{\circ} 08' : Rad. :: (cs. B D \ 42^{\circ} 09') : cs. \angle A D B \ 61^{\circ} 34'.$$

Again;





Again: For the second Operation in the  $\Delta$  ABD, we must compare the  $\angle$ 's B, ADB and the Perpendicular AD, and we shall find that the  $\angle$  B will be *middle Part*; and AD, and Complement of the  $\angle$  ADB, *Extremes Disjunct*: and in the  $\Delta$  ACD, by comparing the Perpendicular and the Term given and required, as before directed, it will be the  $\angle$  C, *middle Part*; and AD and Complement of the  $\angle$  ADC, *Extremes*; wherefore, rejecting the Perpendicular, the Equation is,

$$cs. \angle B + s. \angle ADC = s. \angle ADB + cs. \angle ACD.$$

And by *Analogy*,

As  $cs. \angle B$   $36^{\circ} 08'$ , *middle Part* in the first  $\Delta$ ,

To  $s. \angle ADB$   $61^{\circ} 34'$ , an *Extreme*;

So  $cs. \angle ACD$   $76^{\circ}$ , \* *middle Part* in the second  $\Delta$ ,

To  $s. \angle ADC$   $25^{\circ} 16'$ , an *Extreme*.

Subtract ADC from ADB, there remains the  $\angle D = 46^{\circ} 18'$ .

The Ingenious Mr. Cunn, in the Preface to his Translation of the Elements of Euclid, written in Latin by Dr. John Keil, very justly observes, that most, if not all the Trigonometrical Writers hitherto, (and he instances in three or four of the

\* Note, The  $\angle ACD$  is known, being the Complement of BCD,  $104^{\circ}$ .

foremost,

foremost, viz. Dr. Keil, Dr. Harris, Mr. Caswell, and Mr. Heynes) not only are mistaken in the Solution they have given of the twelfth Case of Oblique Sphericks, by letting fall a Perpendicular, which is undoubtedly the shortest and best Method; but also have left us very much in the dark in their Determination of these two last Cases, and most of the other Cases which follow herein; where they sometimes tell us the *Quæstæ* are ambiguous, and at other times they seem to have spoken as if they were determined; which Doctrine of theirs Mr. Cunn remarks is sometimes true, and sometimes also false: And accordingly (in the Treatise of Trigonometry annexed to those Elements) he gives us certain Directions, to know when the *Quæstæ* are ambiguous, and when not, after having first made the following Observation, viz. " That whereas  
 " every Sine (as we have before shown in this  
 " Work) corresponds to two Arches, to one less  
 " than a Quadrant, and to another, which is the  
 " Supplement of the former, to a Semicircle (a  
 " true Distinction of which of these are to be us'd  
 " being absolutely necessary to be known, before  
 " a proper Solution can be given to such Problems  
 " as these); he asks leave, for Brevity's sake, to  
 " call the Lesser Arch the *Acute Value*, and the  
 " Greater the *Obtuse*; whether the Sine be of an  
 " Angle or a Side". I shall set down his Remarks upon the Cases, as they follow in Order; and then give you his Solution of the matter.

And first, of the two foregoing Cases, where there are given two  $\angle$ 's, B and D, and BC, a Side opposite to D, one of them, to find BD the Side lying between those given  $\angle$ 's, or the third  $\angle$  C; they have told us, that the Sum or Difference of the Bases, or of the Vertical  $\angle$ 's, according as the Perpendicular falls within or without, shall be the Side or  $\angle$  sought; and that it's known whether  
 the

the Perpendicular falls within or without, by the Affection of the given Angles.

Here, says Mr. Cunn, they seem to have spoken, as tho' the *Quæsitum* was always determined, and never ambiguous; for they have here determined, whether the Perpendicular falls within or without; and thereby, whether we are to take the *Sum* or *Difference* of the *Bases*, or of the *Vertical  $\angle$ 's*, for the *Side* or  *$\angle$*  required. Whereas he affirms, that notwithstanding these imaginary Determinations of theirs, the *Quæsitum* here is sometimes ambiguous, and sometimes not; and that too, whether the Perpendicular falls within, or whether it falls without; and accordingly gives the following Rule, whereby we may discover when the *Quæsitæ* are uncertain, and when not.

*Mr. Cunn's Solution of the two Cases above.*

1. When the Perpendicular falls within; i. e. when the given  *$\angle$ 's* are of the same Species; as in the first Example to Case 5.

To the *Acute* Value of *DA*, and also to its *Obtuse* one, add *BA*; and if each of these Sums is less than a Semicircle, then either the *Acute* Value of *DA*, or its *Obtuse* one, added to *BA*, gives the Value of *BD*; which thence is ambiguous. And when only one of these Sums is less than a Semicircle, the *Acute* Value of *DA*, added to *BA*, gives the only Value of *BD*; which then is not ambiguous; tho' in both Varieties the Perpendicular fell within. See the Scheme to Example 1.

2. When the Perpendicular falls without, i. e. when the given  *$\angle$ 's* are of different Species; as in the second Example.

When the *Obtuse* Value of *AC* is less than *AB*, *BC* will be had by subtracting either Value of *AC* from

from AB; and then BC is ambiguous. But when the *Obtuse* Value of AC is not less than AB, the *Acute* Value of AC, taken from AB, leaves the only Value of BC; which therefore is not ambiguous; tho' in both Varieties the Perpendicular fell without. See the Scheme to Exam. 2.

*In the sixth Case, where the same Data are given to find the  $\angle$  C.*

1. Let the Perpendicular fall within; *i. e.* let the given  $\angle$ 's be of the same Species, as in the first Example.

To the *Acute* Value of DCA, and also to its *Obtuse* one, add the  $\angle$  BCA; and if each of these Sums is less than two Right  $\angle$ 's, then either the *Acute* Value of DCA, or its *Obtuse* one added to BCA, gives the Value of BCD; which therefore is ambiguous. And when only one of these Sums is less than two Right  $\angle$ 's, the *Acute* Value of DCA, added to BCA, gives the only Value of BCD; which then is not ambiguous; tho' in both Varieties the Perpendicular fell within. See the Scheme to Exam. 1. of this Case.

2. Let the Perpendicular fall without, *i. e.* let the given  $\angle$ 's be of different Species; as in the second Example.

When the *Obtuse* Value of the  $\angle$  ADC is less than the  $\angle$  ADB, the  $\angle$  BDC may be had by subtracting either Value of ADC from BDA; and then BDC is ambiguous. But when the *Obtuse* Value of ADC is not less than BDA, the *Acute* Value of ADC taken from BDA, gives the single Value of BDC; which therefore is not ambiguous; tho' in both Varieties the Perpendicular fell without. See the Scheme to Exam. 2.

Note here, The Data in both the second Examples are changed, and consequently the *Qualitum* is not the same

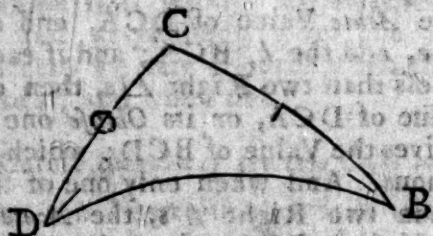


same as in the two first Examples. Which Admonition may be seasonable to a young Beginner.

### CASE VII.

From the same Data as in the two last Cases foregoing; viz.

Two  $\angle$ 's (B and D) and (BC) a Side opposite to one of them (D), to find (CD) the Side opposite to the other.



#### Example.

This and the 10th Case following, wherein there are given and required opposite Parts, will be better resolved by the Axiom underneath, than by any other Method whatever; and therefore I hope the Learner will excuse it, if I omit their Resolution by letting fall a Perpendicular, and only solve them both in the manner there taught; leaving the Trial of the other way to the Learner, if he chuses it; which must certainly be very easy to any one, who has but attended to the foregoing Cases.

#### The Axiom.

In all Spherical Triangles, the Sines of their Sides are in a direct Proportion to the Sines of their opposite Angles; & contra.

That

That is,

As the Sine of a Side : Sine of its opposite  $\angle$  ::  
Sine of another Side : Sine of its opposite  $\angle$ .

And, As the Sine of an  $\angle$  : Sine of its opposite  
Side :: Sine of another  $\angle$  : Sine of its opposite  
Side.

Whence the Solution of the last *Case* above will  
be thus:

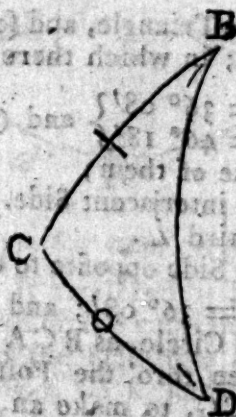
Let  $\angle B = 36^\circ 08'$ ;  $\angle D = 46^\circ 18'$ , and  
 $BC = 30^\circ 00'$ . Then,

As  $s. \angle D 46^\circ 18'$ ,

To  $s. BC 30^\circ 00'$ ;

So  $s. \angle B 36^\circ 08'$ ,

To  $s. CD 24^\circ 04'$ .



Upon this and the 10th *Case* following, Mr. Cunn  
remarks, that all our Trigonometrical Writers  
have told us that the Solutions are ambiguous;  
which, as he observes, is sometimes true, but  
sometimes false; for sometimes the *Quaestum* is  
doubtful, and sometimes not; and when it is not  
doubtful,

doubtful, it is sometimes greater than  $90^\circ$ , and sometimes less; and accordingly gives us this Rule to determine in the Case above.

To the Acute Value of DC, and also to its Obtuse one, add BC; and if each of these Sums are greater than a Semicircle, when the Sum of the  $\angle$ 's BD is greater than two Right  $\angle$ 's; or if they are less than a Semicircle, when the Sum of those  $\angle$ 's is less than two Right  $\angle$ 's, both the Values of DC may be admitted, and then is ambiguous: But when only one of those Sums is  $\left\{ \begin{array}{l} \text{greater} \\ \text{less} \end{array} \right\}$  than a Semicircle, only one Value of DC can be true, viz. the  $\left\{ \begin{array}{l} \text{Obtuse} \\ \text{Acute} \end{array} \right\}$  one, and then is not ambiguous.

To delineate the Triangle, and solve these three Cases Geometrically; in which there are given,

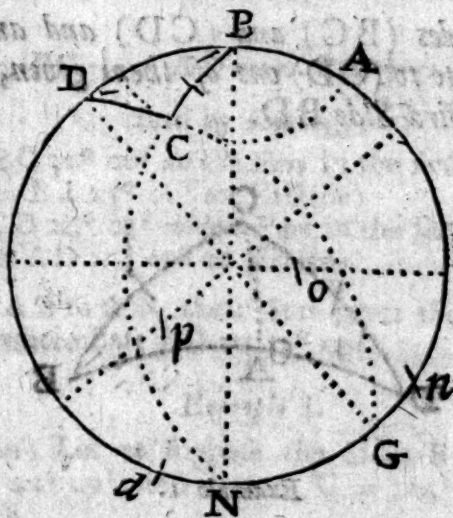
Two  $\angle$ 's  $\left\{ \begin{array}{l} B = 36^\circ 08' \\ D = 46^\circ 18' \end{array} \right\}$  and  $(BC = 30^\circ)$  a Side opposite to one of them;  
to find  $\left\{ \begin{array}{l} BD, \text{ the interjacent Side.} \\ C, \text{ the third } \angle. \\ CD, \text{ the Side opposite to the other.} \end{array} \right.$

Make the  $\angle B = 36^\circ 08'$ ; and  $BC = 30^\circ$ , by drawing a parallel Circle, as BCA; or otherwise, as in Prob. 1. Then thro' the Point C draw the Oblique Circle DCG, to make an  $\angle$  at D =  $46^\circ 18'$ , thus:

A General Rule for drawing of a great Circle thro' a given Point, to make any  $\angle$  with the Primitive.

With the Tangent of the given  $\angle$  [here  $46^\circ 18'$ ] and one Foot in the Centre of the Primitive Circle, make an Arch; and with the Secant of the same  $\angle$ , and one Foot in the given Point [C] cross the former Arch; which Intersection shall give you the Centre of the Oblique Circle [DCG] required to be drawn. Here

Here the Learner is to be put in mind to observe, that the Point given must be so far from the Centre of the Primitive Circle, that the Tangent from thence, and the Secant from the Point given, may intersect each other; otherwise it cannot be done.



*To measure what's required.*

The  $\angle$  at C is measured by laying a Ruler to p and o, the Poles of the containing Sides, and reducing them to the Primitive at d and n, as is taught by *Problem 7. Ch 8.* and has been fully explain'd in the Cases before; and is equal to  $76^\circ$ , whose Complement to  $180^\circ$  is  $104^\circ$ , the  $\angle$  sought.

The Side CD is measured, as in the last; and the Side BD by only applying it to the Chords; and so CD will be found =  $24^{\circ} 04'$ , and BD =  $42^{\circ} 09'$ , being the Measure of the Arch *d.n.*

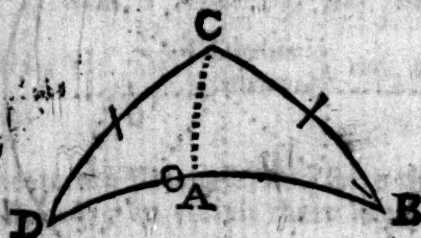


## PROBLEM IV.

Containing the 8th, 9th, and 10th Cases.

## CASE VIII.

Two Sides (BC) and (CD) and an  $\angle$  (B) opposite to (CD) one of them given, to find the third Side BD.



## Example I.

Suppose  $BC = 30^\circ$ ,  $CD = 24^\circ 04'$ , and the  $\angle B = 36^\circ 08'$ .

First, When the Perpendicular falls within, and the  $\triangle$  is thereby reduced into the two  $\triangle$ 's, ABC and ADC, in the former of which there are given the Side BC and the  $\angle$  B, to find the Segment of the Base BA, by the second Case of Rectangle Sphericks, after this manner:

$$\text{cr. } \angle B + \text{Radius} = \text{cr. } BC + \text{r. } BA.$$

Or thus:

$$\text{As cr. } BC \ 30^\circ : \text{Rad} :: (\text{cr. } \angle B \ 36^\circ 08') : \text{r. } BA = 25^\circ 00'.$$

Then

Then, in the next place, if we compare the Base BA, the Hypotenuse BC, and the Perpendicular AC, in the first  $\Delta$ ; the Complement of BC will be *middle Part*, and AB and AC *Extremes Disjunct*; and in the other  $\Delta$ , the Complement of CD will be the *middle Part*, and AD and the Perpendicular AC the *Extremes*. Whence the Equation is,

$$\text{cs. BC} + \text{cs. AD} = \text{cs. BA} + \text{cs. CD.}$$

And by Analogy,

As *cs. BC*  $30^{\circ} 00'$  *middle Part* in the first  $\Delta$ ,

To *cs. BA*  $25^{\circ} 00'$  an *Extreme*;

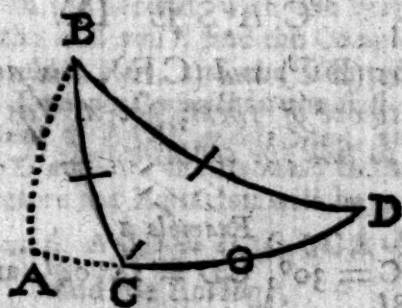
So *cs. CD*  $24^{\circ} 04'$  *middle Part* in the second  $\Delta$ ,

To *cs. AD*  $17^{\circ} 09'$  an *Extreme*.

Add AB to AD, their Sum gives the Quantity of the whole Base BD =  $42^{\circ} 09'$ .

*Example 2.*

Again; Let us suppose the Sides BC =  $30^{\circ}$ , BD =  $42^{\circ} 09'$ , and the  $\angle C = 104^{\circ}$  given, to find CD.



Here the Perpendicular falls without; and the first Operation will be to find the Base AC in the  $\Delta BCD$ ,

$\Delta BCD$ , by the second Case of Rectangle Sphericks, thus:

$$cs. ACB + \text{Radius} = ct. BC \perp t. AC.$$

Or,

$$\text{As } ct. BC 30^\circ : \text{Rad} :: (cs. ACB 76^\circ) : t. AC 7^\circ 57'.$$

Secondly, Let us compare  $BC$ ,  $AC$ , and the Perpendicular  $AB$ , in the  $\Delta ABC$ ; and we shall find the Complement of  $BC$  the *middle Part*, and  $AB$  and  $AC$  *Extremes Disjunct*; and in the  $\Delta ABD$  'twill be Complement of  $BD$  *middle Part*, and  $AB$  and  $AD$  *Extremes*; whence the Equation is,

$$cs. BC + cs. AD = cs. AC + cs. BD.$$

And the Analogy,

As  $cs. BC 30^\circ 00'$ , *middle Part* in the first  $\Delta$ ,

To  $cs. AC 7^\circ 57'$ , an *Extreme*;

So  $cs. BD 42^\circ 09'$ , *middle Part* in the second  $\Delta$ ,

To  $cs. AD 32^\circ 01'$ , an *Extreme*.

Subtra $\hat{c}$ t  $AC$  from  $AD$ , there remains  $CD = 24^\circ 04'$ .

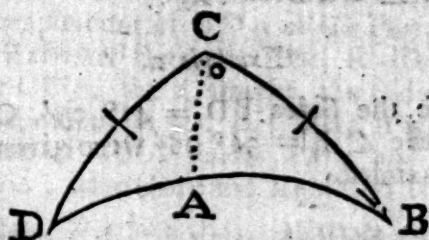
## CASE IX.

Two Sides ( $BC$ ) and ( $CD$ ), and an ( $\angle B$ ) opposite to one of them, given, (as in the last Case) to find the contained  $\angle (C)$ .

Example 1.

Let  $BC = 30^\circ$ ,  $CD = 24^\circ 04'$ , and the  $\angle B = 36^\circ 08'$ .

First,



*First*, When the Perpendicular falls within, and the given  $\Delta$  is divided into two others, namely ABC and ADC; in the first whereof there are granted the Side BC, and the  $\angle$  B, and demanded the Vertical  $\angle$  ACB; which may be found by the third *Case* of Rectangle Sphericks, in this manner:

$$cs. BC + Rad = cs. \angle B + cs. \angle BCA.$$

Or thus:

$$As\ cs.\ \angle B\ 36^{\circ}\ 08' : Rad :: (cs.\ BC\ 30^{\circ}) : cs.\ \angle BCA (= 57^{\circ}\ 42').$$

Compare the Homogenial Terms in both  $\Delta$ 's, viz. BC and the  $\angle$  BCA in the first, and CD and the  $\angle$  ACD in the second, with the Perpendicular AC in both; and you'll find the Complement of the  $\angle$  BCA *middle Part*, and the Complement of BC and AC, *Extremes Conjunct* in the first  $\Delta$ ; and the Complement of ACD *middle Part*, and Complement of CD and AC the *Extremes* in the second  $\Delta$ ; and therefore the Equation will be

$$cs. BC + cs. \angle ACD = cs. \angle BCA + cs. CD.$$

And the Analogy,

As  $cs. BC\ 30^{\circ}$  an *Extreme* in the first  $\Delta$ ,

To  $cs. \angle BCA\ 57^{\circ}\ 42'$  *middle Part*;

So  $cs. CD\ 24^{\circ}\ 04'$  an *Extreme* in the second  $\Delta$ ,

To  $cs. \angle ACD\ 46^{\circ}\ 18'$  *middle Part*.

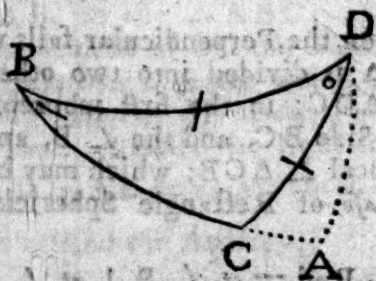
To



To this  $\angle ACD$  add the  $\angle BCA$ , and you have the whole  $\angle BCD = 104^\circ$ .

*Example 2.*

Suppose the Sides  $BD = 42^\circ 09'$ ,  $CD = 24^\circ 04'$ , and the  $\angle B = 36^\circ 08'$  were given, and the  $\angle D$  demanded.



Here, because the  $\angle$ 's B and C at the Base are of a different Affection, therefore the Perpendicular falls without; and, by the aforementioned Rule, it must fall from the End of the given Side CD, and subtend the given Angle B, as in the Scheme above.

And then, in the  $\triangle ABD$ , we have given the Side BD and  $\angle B$ , to find the Vertical  $\angle BDA$ , by the third Case of *Rectangle Sphericks*, thus:

$$\text{cs. } BD + \text{Rad} = \text{cs. } \angle B + \text{cs. } \angle BDA.$$

Or,

$$\text{As cs. } \angle B 36^\circ 08' : \text{Rad} :: (\text{cs. } BD 42^\circ 09') : \text{cs. } \angle BDA = 61^\circ 34'.$$

Secondly, Having found the  $\angle BDA$ , we next enquire the  $\angle CDA$ ; in order whereunto we must compare the Side BD, the  $\angle BDA$ , and Perpen-

Perpendicular AD in the  $\triangle ABD$ , and we shall find the Complement of BDA *middle Part*, and BD and AD *Extremes Conjoint*: And so likewise by comparing CDA, the  $\angle$  sought, the Side CD, and the Perpendicular, in the  $\triangle CDA$ ; it will be the Complement of CDA *middle Part*, and CD and DA *Extremes*; wherefore the Equation is,

$$ct. BD + cs. \angle CDA = cs. \angle BDA + ct. CD.$$

And the *Analogy*,

As *ct. BD*  $42^{\circ} 09'$ , an *Extreme* in the first  $\triangle$ ,  
 To *BDA*  $61^{\circ} 34'$ , *middle Part*;  
 So *ct. CD*  $24^{\circ} 04'$ , an *Extreme* in the second  $\triangle$ ,  
 To *cs. CDA*  $15^{\circ} 13'$ , *middle Part*.

Subtra& the  $\angle$  CDA from the whole  $\angle$  BDA, there will remain the  $\angle$  BDC  $= 46^{\circ} 21'$ .

— Upon these two Cases, Mr. *Cunn* makes the following Remark, *viz.*

That all the Writers of Trigonometry that he has met with, who have undertaken the Solution thereof, as well as the 5th and 6th Case, by letting fall a Perpendicular, have told us, that the *Sum* or *Difference* of the *Vertical*  $\angle$ 's, or of the *Bases*, shall be the *Angle* or Side demanded, according as the Perpendicular falls *within* or *without*; which cannot be known, unless the *Species* of that unknown  $\angle$ , which is opposite to a given Side, be first known.

Here, he observes, they leave us first to calculate that unknown Angle, before we shall know whether we are to take the *Sum* or the *Difference* of the *Vertical Angles* or *Bases*, for the sought *Angle* or *Base*: and in the Calculation of that *Angle*, have left us in the dark, as to its *Species*; as appears by his Observations on the 5th and 6th Cases afore set down.

The

The Truth is, says he, the *Quaestum* here, as well as in those two Cases, is sometimes doubtful, and sometimes not; when doubtful, sometimes each Answer is *less than 90°*, sometimes each is *greater*; but sometimes one *less*, and the other *greater*, as in the two last mentioned Cases. When it is not doubtful, the *Quaestum* is sometimes *greater than 90°*, and sometimes *less*. All which Distinctions, he observes, may be made without another Operation, or the Knowledge of the *Species* of that unknown Angle, opposite to a given Side; or, which is the same thing, the falling of the Perpendicular within or without, and in order hereunto sets down the following Rule.

*Mr. Cunn's Solution of these Cases.*

*See the first Example to Case 8.*

*First*, We may observe, that the *Species* of DA, the Term found by the second Operation is always known; for it is of the same Affection with the given  $\angle B$ , when DC is *less than a Quadrant*: but of a different Affection with the  $\angle B$ , when DC is *greater than a Quadrant*. And,

If AD be *less than AB*, and also the Sum of AD and AB *less than a Semicircle*; then AD, either added to, or subtracted from AB, will give the Value of BD; which therefore is *ambiguous*.

But if AD be *not less than AB*, or if their Sum be *not less than a Semicircle*; then their Sum in the former, and their Difference in the latter Variety, shall give one single Value of BD; which then is *not ambiguous*.

*His Solution of the 9th Case.*

*See the first Example.*

Here we may observe, that the *Species* of the  $\angle ACD$ , the Term found by the second Operation,

tion, is known; for it is of the same kind with the given  $\angle B$ , when  $DC$ , the Side opposite thereunto, is less than a Quadrant; but of a different kind with the  $\angle B$ , when  $DC$  is greater than a Quadrant. And

*If  $DCA$  be less than  $BCA$ , and the Sum of  $DCA$  and  $BCA$  less than two Right  $\angle$ 's; then  $DCA$  either added to, or subtracted from  $BCA$ , gives the  $\angle BCD$ ; which therefore is ambiguous.*

*If  $DCA$  be not less than  $BCA$ , or the Sum of  $DCA$  and  $BCA$  not less than two Right  $\angle$ 's; then their Sum in the former, and their Difference in the latter Variety, shall give the single Value of  $BCD$ ; which then is not ambiguous.*

### CASE X.

From the same Data as in the two last foregoing Cases, viz.

*Two Sides  $BC$ ,  $DC$ , and an  $\angle B$ , opposite to  $DC$  one of those Sides, to find  $D$  the  $\angle$  opposite to the other.*

*Example.*

Let  $BC = 30^\circ$ ,  $DC = 24^\circ 04'$ , and the  $\angle B = 36^\circ 08'$ , as before.

Then the Solution of this Case will, by the last mentioned *Axiom*, be thus:

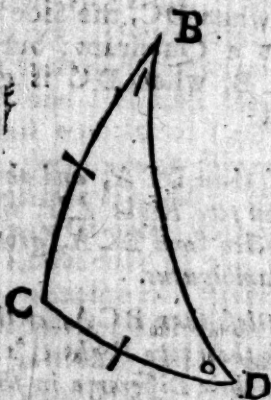
As  $\therefore$ ,  $CD\ 24^\circ 04'$ ,

To  $\therefore$ ,  $\angle B\ 36^\circ 08'$ ;

So  $\therefore$ ,  $BC\ 30^\circ 00'$ ,

To  $\therefore$ ,  $\angle D\ 46^\circ 18'$ .





See Mr. *Cunn*'s Remark upon this Case in the 7th Case; and his Solution hereof, as follows:

To the *Acute* Value of the  $\angle D$ , and also to its *Obtuse* Value, add the  $\angle B$ ; and if each of these Sums is *greater* than two Right  $\angle$ 's, when the Sum of the Sides is greater than a Semicircle; or if each of them be *less* than two Right  $\angle$ 's, when the Sum of the Sides is *less* than a Semicircle, both the Values of  $D$  may be admitted, and consequently  $D$  is ambiguous:

But when only one of those Sums is *greater* or *less* than two Right  $\angle$ 's, only one Value of  $D$  is true, viz. the  $\left\{ \begin{array}{l} \text{Obtuse, when greater,} \\ \text{Acute, when less,} \end{array} \right\}$  and then not ambiguous.

To delineate the Triangle, and solve these three Cases Geometrically; in which there are given

Two Sides  $\left\{ \begin{array}{l} BC = 30^{\circ} 00' \\ CD = 24^{\circ} 04' \end{array} \right\}$  and  $B = 36^{\circ} 08'$ ,  
an  $\angle$  opposite to  $CD$  one of them,



Observe, If this last Circle had been drawn through the other Point, where the Parallel cuts the Oblique Circle BCH; then the  $\angle C$  would have been Acute, whereas now it is Obtuse.

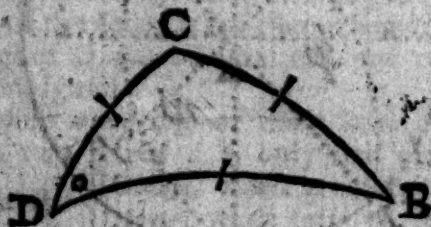
To measure the Parts required.

The Side BC is measured by Prob. 6. and the  $\angle$ 's by Prob. 7. as in the Cases before. And so BC will be found =  $30^\circ 00'$ , the  $\angle BCD =$  (br)  $104^\circ$ , or  $76^\circ 00'$ , according to what it is, whether Obtuse or Acute; and the  $\angle BDC =$  (An)  $46^\circ 18'$ .

# PROBLEM V.

## CASE XI.

All the Sides, viz. BC (=  $30^\circ$ ) CD (=  $24^\circ 04'$ ) and BD (=  $42^\circ 09'$ ) given; to find an  $\angle$ , as D.



For resolving of this and the following Case, the Catholick Proposition of the Lord *Nepier* is not of itself sufficient, without some Preparation being made, which makes it both tedious and troublesome to a young Beginner; and therefore I chuse to omit their Solutions after the Method which he has laid down, and instead thereof shall insert one which

which is much shorter and easier ; and the rather, because these two Cases are the most useful in *Astronomy, Navigation, Dialling, &c.* of any ; for the one finds the *Hour*, and the other the *Azimuth*, both which are exceeding necessary on many Occasions.

*The Method which I propose is this :*

Add the three Sides together, and from half their Sum deduct the Side opposite to the enquired Angle, noting the Difference, and then proceed thus :

Set down the *Arithmetical Complements* of the *Sines* of the two Sides, comprehending the  $\angle$  sought, and also the *Sines* of the half Sum and Difference, and add them all up into one *Sum* ; the half whereof is the *Sine* of an *Angle*, which being doubled, gives the Angle required.

An Example will make it very plain.

|                          |                       |
|--------------------------|-----------------------|
| Side BC $30^{\circ} 00'$ | <i>Arithm. Compl.</i> |
| Side CD $24^{\circ} 04'$ | 0.3895535             |
| Side BD $42^{\circ} 09'$ | 0.1732297             |

Sum  $96^{\circ} 13'$

|                                                |               |
|------------------------------------------------|---------------|
| $\frac{1}{2}$ Sum $48^{\circ} 06' \frac{1}{2}$ | <i>Sines.</i> |
|                                                | 9.8717548     |

Side BC  $30^{\circ} 00'$

Diff.  $18^{\circ} 06' \frac{1}{2}$

19.9268463

$\frac{1}{2}$  Sum is 9.9634231  $\frac{1}{2}$

The *Sine* of  $66^{\circ} 50' \frac{1}{2}$ , which doubled and subtracted from 180, gives  $46^{\circ} 19'$ , the Quantity of the  $\angle$  D required.





*The Instrumental Performance of this Case by the Line of Sines is after this manner.*

As Radius : Sine of one of the containing Sides (CD  $24^{\circ} 04'$ ) :: Sine of the other containing Side (BD  $42^{\circ} 09'$ ) : a fourth Sine, viz.  $15^{\circ} 53'$ .

Then, as that fourth Sine ( $15^{\circ} 53'$ ) : Sine of the half Sum of the three Sides ( $48^{\circ} 06'$ ) :: Sine of the Difference ( $18^{\circ} 06'$ ) : another Sine ( $57^{\circ} 40'$ ); divide the Space between  $57^{\circ} 40'$  and  $90^{\circ}$  into two equal Parts, the middle Point between them will fall upon  $66^{\circ} 50' 30''$ ; which being doubled and subtracted from  $180^{\circ}$ , gives  $46^{\circ} 19'$  the  $\angle D$ , as before.

## PROBLEM VI.

### CASE XII.

*All the Angles given to find the Sides.*

Let the  $\angle$ 's be the same as above, and CD the Side sought.



That

That the common Solution of this Case is false, Mr. *Cunn* has demonstrated in the Treatise aforementioned, and given us a true one, which in short is this:

Change one of the  $\angle$ 's, adjacent to the Side sought, into its Supplement; and then work with the Measures of the Angles, as tho' they were Sides, as in the last Case, and the Result will be the Side sought.

*Example.*

|                                          |                                |
|------------------------------------------|--------------------------------|
| $\angle B$ , $36^{\circ} 08'$            | <i>Comp'l. Arithm.</i>         |
| $\angle D$ , $46^{\circ} 18'$            | 0.1408814                      |
| Supplement $\angle C$ , $76^{\circ} 00'$ | 0.0130959                      |
| Sum $158^{\circ} 26'$                    | <i>Sines.</i>                  |
| $\frac{1}{2}$ Sum $79^{\circ} 13'$       | 9.9922626                      |
| $\angle B$ $36^{\circ} 08'$              |                                |
| Difference $43^{\circ} 05'$              | 9.8344579                      |
|                                          | 19.9806978                     |
|                                          | $\frac{1}{2}$ Sum is 9.9903489 |

The Sine Complement of  $12^{\circ} 02'$  half the Side CD, which being doubled, gives the Side CD  $24^{\circ} 04'$ , as was required.

The Geometrical Effect of this Case is the same, in Effect, as the last; for if the  $\angle$ 's be changed into Sides, as is shewed before, and a Triangle be made therewith, in all respects as in the last Case, it will answer the Conditions required.

And thus have I finished the whole Doctrine of Trigonometry, both Plain and Spherical, three several ways, as I intended; and have annexed the several Varieties to each Case; and also have set down and explain'd such general Rules and Axioms,

as I thought expedient for the better illustrating the same: In the doing whereof, I have endeavour'd to use the utmost Clearness, both in the *Rule* and in the *Example*, and hope I have made it plain and intelligible to every mean Capacity. But before I shut up this Chapter, I shall exhibit to the Learner, in one View, all the Cases of Right-angle Sphericks, with their Proportions and Equations; and also the first Ten Cases of Oblique Sphericks, briefly comprehended in two small Tables, as follows:

*A Synopsis; or brief View of the sixteen Cases of Rectangled Sphericks, with their Analogies and Equations.*

Observe, That the *middle Part*, whether given or demanded, is included within this ( ) Mark.



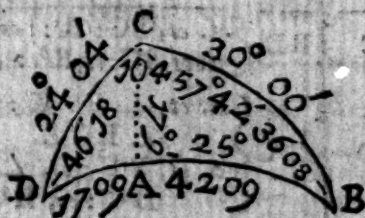
T A B L E I.

| Ca. | Given.           | Req.       | Equations and Analogies.                                                                               |
|-----|------------------|------------|--------------------------------------------------------------------------------------------------------|
| 1   | A C $\angle$ A   | (BC)       | $s, BC + R = s, AC + s, \angle A$ .<br>Rad: $s, AC :: s, \angle A : s, (BC)$                           |
| 2   | A C $\angle$ (C) | BC         | $cs, \angle C + R = t, BC + ct, AC$ .<br>$ct, AC : R :: cs, \angle (C) : t, BC$ .                      |
| 3   | (AC) $\angle$ A  | $\angle$ C | $cs, AC + R = ct, \angle A + ct, \angle C$ .<br>$ct, \angle A : R :: cs, (AC) : ct, \angle C$ .<br>Ca. |



| <i>Ca.</i> | <i>Given.</i>   | <i>Req.</i> | <i>Equations and Analogies.</i>                                                              |
|------------|-----------------|-------------|----------------------------------------------------------------------------------------------|
| 4          | $AC(BC)$        | $\angle A$  | $s, BC + R = s, AC + s, \angle A.$<br>$s, AC : R :: s(BC) : s, \angle A.$                    |
| 5          | $ACBC$          | $\angle(C)$ | $ct, AC + t, BC = R + cs, \angle C.$<br>$R : ct, AC :: t, BC : cs, \angle(C)$                |
| 6          | $(AC)BC$        | $AB$        | $cs, AC + R = cs, BC + cs, AB.$<br>$cs, BC : R :: cs(AC) : cs, AB.$                          |
| 7          | $BC \angle A$   | $(AB)$      | $t, BC + ct, \angle A = R + s, AB.$<br>$R : ct, \angle A :: t, BC : s, (AB)$                 |
| 8          | $BC \angle (A)$ | $\angle C$  | $cs, \angle A + R = cs, BC + s, \angle C.$<br>$cs, BC : R :: cs, \angle(A) : s, \angle C.$   |
| 9          | $(BC) \angle A$ | $AC$        | $s, BC + R = s, \angle A + s, AC.$<br>$s, \angle A : s(BC) :: R : s, AC.$                    |
| 10         | $(AB) \angle A$ | $BC$        | $s, AB + R = ct, \angle A + t, BC.$<br>$ct, \angle A : R :: s, (AB) : t, BC.$                |
| 11         | $AB \angle A$   | $\angle(C)$ | $cs, AB + s, \angle A = R + cs, \angle C.$<br>$R : cs, AB :: s, \angle A : cs, \angle(C)$    |
| 12         | $AB \angle (A)$ | $AC$        | $cs, \angle A + R = t, AB + ct, AC.$<br>$t, AB : R :: cs, \angle(A) : ct, AC.$               |
| 13         | $(AB)BC$        | $A$         | $s, AB + R = t, BC + ct, \angle A.$<br>$t, BC : s, (AB) :: Rad : ct, \angle A.$              |
| 14         | $ABBC$          | $(AC)$      | $cs, BC + cs, AB = R + cs, AC.$<br>$Rad : cs, BC :: cs, AB : cs, (AC)$                       |
| 15         | $\angle(A)C$    | $BC$        | $cs, \angle A + cs, \angle C = R + cs, BC.$<br>$s, \angle C : cs, \angle(A) :: R : cs, BC.$  |
| 16         | $\angle AC$     | $(AC)$      | $ct, \angle A + ct, \angle C = R + cs, AC.$<br>$R : ct, \angle A :: ct, \angle C : cs, (AC)$ |

TABLE



# TABLE II.

Of the first Ten Cases of Oblique Sphericks.

| Ca. | Given.                   | Req.       | Analogies.                                                                                                                                                             |
|-----|--------------------------|------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1   | BC BD                    | $\angle D$ | $\left\{ \begin{array}{l} 1. ct, BC : R :: cs, \angle B : t, BA. \\ 2. s, BA : ct, \angle B : s, AD : ct, \angle D. \end{array} \right.$                               |
| 2   | $\angle B$               | CD         | $\left\{ \begin{array}{l} 1. ct, BC : R :: cs, \angle B : t, AB. \\ 2. cs, AB : cs, BC :: cs, AD : cs, CD. \end{array} \right.$                                        |
| 3   | $\angle B$<br>$\angle C$ | CD         | $\left\{ \begin{array}{l} 1. ct, \angle B : cs, BC :: R : ct, \angle BCA. \\ 2. cs, \angle BCA : ct, BC :: cs, \angle ACD : ct, DC. \end{array} \right.$               |
| 4   | BC                       | $\angle D$ | $\left\{ \begin{array}{l} 1. ct, \angle ABC : cs, BC :: R : ct, \angle ACB. \\ 2. s, \angle ACB : cs, \angle ABC :: s, \angle ACD : cs, \angle D. \end{array} \right.$ |
| 5   | $\angle B$<br>$\angle D$ | BD         | $\left\{ \begin{array}{l} 1. ct, BC : R :: cs, \angle B : t, AB. \\ 2. ct, \angle B : s, AB :: ct, \angle D : s, AD^* \end{array} \right.$                             |
| 6   | BC                       | $\angle C$ | $\left\{ \begin{array}{l} 1. ct, \angle B : R :: cs, BC : ct, ACB \\ 2. cs, \angle B : s, \angle ACB :: cs, \angle D : s, ACD. \end{array} \right.$                    |
| 7   | —                        | CD         | $s, \angle D : s, BC :: s, \angle B : s, CD.$                                                                                                                          |

See the Remarks upon these three Cases.

\* Then  $AD + AB = BD.$

8 BC

| Ca. | Given.     | Req.       | Analogies.                                                                                                                                  |
|-----|------------|------------|---------------------------------------------------------------------------------------------------------------------------------------------|
| 8   | B C        | BD         | $\left\{ \begin{array}{l} 1. ct, BC:R::cs, \angle B:t, BA. \\ 2. cs, BC:cs, BA::cs, CD:cs, AD* \end{array} \right.$                         |
| 9   | C D        | $\angle C$ | $\left\{ \begin{array}{l} 1. ct, \angle B:R::cs, BC:ct, \\ \angle BCA. \\ 2. ct, BC:cs, \angle BCA::ct, \\ DC:cs, ACD. \end{array} \right.$ |
| 10  | $\angle B$ | $\angle D$ | $s, CD:s, \angle B::s, BC:s, \angle D.$                                                                                                     |

*See also the Remarks upon these Cases.*

\* Then  $AD + AB = BD.$

To avoid the Trouble, and oftentimes Confusion to a Learner, in turning over the Tables of *Logarithmical Sines, Tangents, &c.* I have here set down the *Sines, Co-sines, Tangents and Co-tangents* of all the *Sides and Angles* of the Triangle made use of both here, and also in *Rectangled Sphericks*; as followeth.

### 1. Of Rectangled Sphericks.

| Sides and Angl. | Meas.   | Sine.    | Co-sine. | Tangent.  | Co-tang.  | Compl.  |
|-----------------|---------|----------|----------|-----------|-----------|---------|
| AC              | 46° 31' | 9.860682 | 9.837679 | 10.023003 | 9.976997  | 43° 29' |
| BC              | 27° 48' | 9.668746 | 9.946738 | 9.722008  | 10.277991 | 62° 12' |
| AB              | 38° 56' | 9.798247 | 9.890911 | 9.907336  | 10.092664 | 51° 04' |
| $\angle A$      | 40° 00' | 9.808067 | 9.884254 | 9.923812  | 10.076188 | 50° 00' |
| $\angle C$      | 60° 00' | 9.937531 | 9.698970 | 10.238561 | 9.761439  | 30° 00' |

2. Of

2. Of Oblique Sphericks.

| Sides and Angl. | Meas.   | Sine.    | Co-sine. | Tangent.  | Co-tang.  | Compl.  |
|-----------------|---------|----------|----------|-----------|-----------|---------|
| BC              | 30° 00' | 9.698970 | 9.937531 | 9.761439  | 10.238561 | 60° 00' |
| BD              | 42 09   | 9.826770 | 9.870047 | 9.956723  | 10.043277 | 47 51   |
| DC              | 24 04   | 9.610446 | 9.960505 | 9.649942  | 10.350058 | 65 56   |
| ∠B              | 56 08   | 9.778866 | 9.907112 | 9.863385  | 10.136615 | 34 52   |
| ∠C              | 104 00  | 9.986964 | 9.83675  | 10.603229 | 9.396770  | 75 00   |
| ∠D              | 46 18   | 9.859110 | 9.839404 | 10.019714 | 9.980285  | 43 42   |
| BA              | 25 00   | 9.625948 | 9.957276 | 9.668674  | 10.331326 | 65 00   |
| DA              | 17 09   | 9.469637 | 9.980247 | 9.489390  | 10.510610 | 72 51   |
| BCA             | 57 42   | 9.926991 | 9.787447 | 10.119913 | 9.880086  | 32 18   |
| ACD             | 46 18   | 9.859110 | 9.839404 | 10.019714 | 9.980285  | 43 42   |
| CA              | 17 09   | 9.469637 | 9.980247 | 9.489390  | 10.510610 | 72 51   |

I did here intend to have set down several Problems relating to *Geography* and *Astronomy*, to be perform'd by the Use of a *plain Scale* and *Compasses* only; but finding the Work to swell under my Hands beyond what I first designed, I shall omit them till some farther Opportunity; when, if I meet with Encouragement in this, I may publish them, and some other Mathematical Performances, which will be both useful and entertaining; and only present the Learner, in this place, with one very curious Problem, which will solve several of the principal Propositions in *Astronomy*, and that with a very few Lines, in one *Diagram* or *Scheme*, as you may see by what follows.

E X

A a

A P a o-



## A PROBLEM,

*Shewing how to resolve the two last and most difficult Cases of Oblique  $\angle^d$  Sphericks by a Plain Scale and Compasses, without any Labour or Trouble.*

The Day of the Month, and the Sun's Altitude or Height being given, how to find the Hour and Azimuth; both which are exceeding useful in many Propositions of Astronomy, Navigation, Dialling, &c.

Let the Day of the Month be the 25th of April, 1730, and the Sun's Altitude  $34^\circ$  in the Morning. Describe the Semicircle  $WOZ$ , in Fig. 4. for the Meridian. Make  $HO$  the Horizon, and draw  $EC$ , making an  $\angle$  of the Co-altitude  $38\frac{1}{2}$  Degrees to represent the Equator. Seek the Declination of the Sun in some Tables for that Purpose, which you will find near  $16^\circ\frac{1}{2}$  Northward; and set it from  $E$  to  $D$ . Draw  $DR$  for the Path of the Sun that Day parallel to  $EC$

the Equator. Then set the Altitude  $34^\circ$  from  $H$  to  $A$ , draw  $AL$  parallel to  $HO$  the Horizon; and where it intersects  $DR$ , as at  $\odot$ , is the Place of the Sun for that Day. Now to find the Hour and Azimuth, draw the Line  $NC$  at right  $\angle$ 's with the Equator  $EC$ , and the Line  $CZ$  perpendicular to  $OH$ , so will you obtain both; for the Distance  $6\odot$  is the Sun's Hour from 6; that is his Hour after 6 in the Morn, or before 6 in the Afternoon; and the Extent  $FO$  is the Sun's Azimuth from East in the Morn, or from West in the Afternoon.

N. B.

N. B. In the Winter half Year, when the Sun has South Declination, it must be set downward from E toward H, that is, to W; and then you must produce the Line NC below the Horizon down to S, that so you may measure the Hour from S to 6.

And thus is your Scheme prepared to find, besides the Hour and Azimuth, the Length of the Day, which is the Semidiurnal Arch DR; the Sun's Ascensional Difference, which is the Distance 6R; his Amplitude, which is CR; his Azimuth from East or West at six, which is T6; his Altitude at East and West, which is VC; his Meridian Altitude, which is the Arch DH; and his Azimuth from East or West, at rising or setting, which is the Line CR; and may all be measured by the Directions before given, in Chap. 8.



## CHAP. XI.

*Of the Nature of Proportion, and the chief Grounds and Reasons for varying of the same,*

THE Knowledge of Proportion is of such universal Use in the Mathematicks, and so needful for a Beginner to be acquainted withal, that I could by no means content myself without affording it a Place in this Work. And I must confess, I have often wondred how it comes to pass, that most of our Writers of what they term Introductions to Mathematicks, have either

wholly omitted it; or treated it in such a manner, as was nor at all likely should be of service to a young Learner. I have therefore determined to make an Attempt that way, and try if we cannot, by a familiar Example, explain the Mysteries of Proportion, so as to be easily comprehended by an ordinary Apprehension, which pass for the most difficult Things in Geometry, as unquestionably they are the most important. But that I may not grasp at too much, and thereby exceed my prescribed Limits, I shall confine myself to speak only of such kind of Proportion, as falls naturally within the Compass of my present Design in this Undertaking; which I shall endeavour to set in the clearest Light I am able. Let it then, in this place, suffice to give the following Definition of Proportion.

## §. 1.

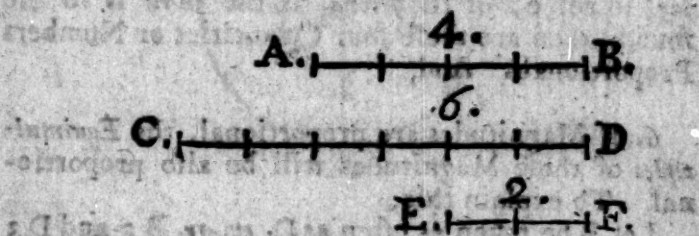
1. *Proportion* (in Mathematicks) is a certain *Similitude, Likeness, or Agreeableness* of *Ratio's*, when several Quantities or Numbers are compared one to another, with respect to their *Greatness* or *Smallness*: Or it is the *Respect and Comparison* which one Magnitude has to another of the same kind; and therefore must consist of two Terms at the least, *viz.* the *Antecedent*, or that which goeth before; the *Consequent*, or that which follows after.

*Note, When we consider one Quantity in respect of another, to see what Magnitude it hath in Comparison of that other; the Magnitude so found is called its Ratio or Reason, tho' I think it would be more intelligible if it were call'd Comparison.*

2. *Proportion*, or the *Respect of one Quantity to another*, is either *Rational* or *Irrational*.

*Rational*

*Rational Proportion* is the Respect of one Quantity to another, which is commensurable to it; that is, when both the Quantities have the same common Measure by which both may be exactly measured; as the Proportion of the Line AB, di-



vided into four equal Parts, to the Line CD, which contains 6 of those equal Parts, is Rational; because the Line EF, which contains two of those Parts, will exactly measure both; and in all such Cases as this, the Quantities themselves have the same Proportion as one Number to another.

*Irrational Proportion* is betwixt two Quantities of the same kind which are incommensurable; that is, have no common Measure. As the Proportion of the Side of a Square to its Diagonal; for there never was any Measure found yet, nor perhaps never will, that would exactly measure both.

3. *Analogy*, or, as some term it, *Proportionality*, is a Similitude of Proportions, and consists of three Terms at least. 'Tis frequently used in common Discourse for the word Proportion.

4. When three Magnitudes are Proportionals, the first is said to have to the third, a duplicate Ratio to what it has to the second. But when four Magnitudes are Proportionals, the first shall have a

A. a. 3.

triplicate



triplicate Ratio to the fourth, of what it has to the second; and so always one more, in order as the Proportionals shall be extended.

5. If when we find that of four *Quantities* or *Numbers*, the *first* hath as much Magnitude (or *is as big*) in respect of the *second*, as the *third* is to the *fourth*, then are those four *Quantities* or *Numbers* *Proportionals*. And,

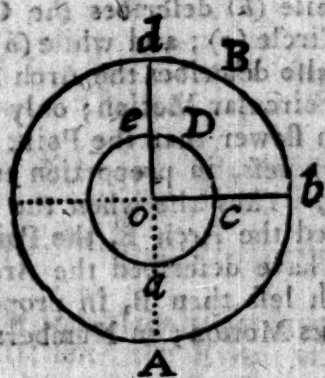
6. If Magnitudes are proportional, the *Equimultiples* of those Magnitudes will be also proportional. To explain this:

Let B be taken as often as D, *ex. gr.* B 3 and D 3 times, we may conclude that  $B : D :: 3 B : 3 D$ , or as 10 B : 10 D, also  $12 \frac{1}{4} B : 12 \frac{1}{4} D$ . And so on in whatever Proportion the two Magnitudes B and D are multiplied, so they are multiplied equally, or that you take one as often as you take the other. So also if B be divided in the same manner as D, *i. e.* if you take a 4th, 10th, or any other Part of B, and the like of D; then will these Parts be proportional to their Wholes; for as  $B : D :: \frac{1}{4} B : \frac{1}{4} D$  or  $\frac{1}{10} B : \frac{1}{10} D$ .

These being premised, I shall now endeavour to explain the Nature of Proportion by such an Example, as will, I hope, make it plain and evident to the meanest Understanding. And first,

Let us imagine the Circle  $b A d$  to be described by the Motion of the Radius  $ob$ , round the Centre  $o$ ; and at the same time let the Circle  $c, a, e$ , be described by the Motion of the Point  $(c)$  in the Radius  $ob$ . Suppose also the Radius  $od$  to be drawn at right Angles to  $ob$ . Let the Arc  $d B d$  be called B, and the Arch  $e D e$  be called D. Let A signify the whole outward Circle, and  $(a)$  the whole inward one.

Then,



Then, if we compare the whole Circle A with its Arch B, and the whole Circle (a) with its Arch D, we shall find that the Circle A is just as big, in respect of the Arch B, as the inward one (a) is in respect of the Arch D; for if B be a 4th (or any other) Part of the Circle A, D also will be a 4th, or the same proportional Part of its Circle (a); and therefore, as  $A : B :: a : D$ , or  $24 : 6 :: 8 : 2$ . All which is very obvious.

But if you would change the Order of the Terms, and compare B with A, and D with (a); you will find plainly that  $B : A :: D : a$ ; which is called *Inverse Proportion*.

Again; if you change them, so as to compare Antecedent with Antecedent, and Consequent with Consequent, you will find by *Alternate Proportion*, that  $A : a :: B : D$ . And this is very plain; for if the whole Circle A be double, triple, or in any other Proportion, to the Circle a, the Arch B will be double, triple, or in the same Proportion to the Arch D; since aliquot Parts are as their Wholes. And this, I say, must needs appear very plain, because the two Circles A and (a) are described by the Motion of the Radius ocb; in such sort,

fort, that while (*b*) describes the Circle *A*, *c* describes the Circle (*a*); and while (*b*) describes the Arch *B*, (*c*) also describes the Arch *D*, and this by one common circular Motion; only the Point (*c*) moving much slower than the Point (*b*), describes a Circle much less, in proportion to the Slowness of its Motion. Thus also when the Point (*b*) shall have described the Arch *B*, the Point (*c*) in like manner will have described the Arch *D*; which will be much less than *B*, in proportion to the Slowness of its Motion; in Numbers, as 24 : 8 :: 6 : 2.

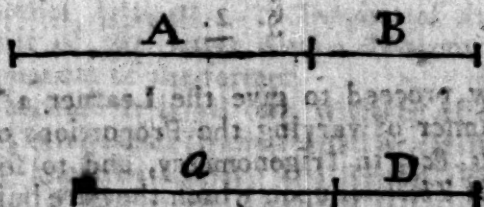
Next, if we compare the Differences between the Antecedents and Consequents, with their Consequents: *As for Instance*; *A* less *B* with *B*, and (*a*) less *D* with *D*; we shall find they also are proportional; and that  $A \text{ less } B : B :: (a) \text{ less } D : D$ ; i. e.  $18 : 6 :: 6 : 2$ .

For 'tis manifest, that the Arch *bAd* (which is  $A - B$ ) is to *B* as the Arch *cae* (which is  $(a) - D$ ) is to *D*. And this is called *Proportion by Division*.

And in the fifth place, if we add the Antecedents and Consequents together, we shall find that  $A + B$  is to *B* ::, as  $(a) + D$  is to *D*. Which is called *Composition of Proportion*. In Numbers,  $30 : 6 :: 10 : 2$ .

Lastly, If we compare the Antecedent with the Difference of the Terms, i. e. with the *Excess*, by which the *Antecedent* exceeds the *Consequent*; then are they also proportional: As, if there be the same Proportion of *AB* to *B*, as of *aD* to *D*, as above, we may thence conclude, that there is the same Proportion of *AB* to *A*, as of *aD* to *a*. And this is called *Conversion of Proportion*.

But this may perhaps want a farther Explication to make it easy to a Beginner; which take as follows:



Let there be four Magnitudes Proportionals, viz.  $AB, B, aD, D$ ; since the Excess of  $AB$ , the first Antecedent, above  $B$  its Consequent, is  $A$ ; and the Excess of  $aD$ , the second Antecedent, above  $D$  its Consequent, is  $a$ . If we compare them according to this Definition, it's plain the Analogy will be thus; as,  $AB : A :: aD : a$ .

To illustrate the same by Numbers, let us take these four —  $9 : 6 :: 12 : 8$ , wherein either Proportion is *Sesquialteral*; that is, when each Antecedent contains its respective Consequent once and a half; so 9 contains 6, once and a half, and 12 contains 8 in the same manner; the Excess between 9 the first Antecedent, and 6 its Consequent, is 3; and between 12 and 8, the second Antecedent and Consequent, is 4; therefore by Conversion of Proportion, it must be as  $9 : 3 :: 12 : 4$ .

See in one View all these different Manners of Argumentation by Proportion,

1. As,  $A : B :: a : D$ , therefore
- By Inverse Prop. 2. As,  $B : A :: D : a$ .
- By Alternate Prop. 3. As,  $A : a :: B : D$ .
- By Division of Prop. 4. As,  $A - B : B :: a - D : D$ .
- By Compos. of Prop. 5. As,  $A + B : B :: a + D : D$ .
- By Conyer. of Prop. 6. As,  $AB : A :: aD : a$ .



## §. 2.

I now proceed to give the Learner a Taste of the Manner of varying the Proportions of *Sines*, *Tangents*, &c. in Trigonometry, and to set down the chief *Theorems* upon which they are built; and because I find them briefly collected to my hand from *Pitiscus*, *Norwood*, and others, and explain'd by Mr. *Hewney*, I shall give it you in his own Words as follows, with leave only to put down the Proportions symbolically, and not in Terms at length, as is there done.

*To vary Proportions.*

## THEOREM I.

" The Radius is a mean Proportional between  
" the *Tangent* of an Arch, and the *Tangent* of its  
" Complement; *that is*,

" As  $1$ ,  $TS$  : Radius  $TC$  : : Radius  $CF$  :  $u$ ,  $FN$ .  
See Fig. 2.

## THEOREM II.

" The Radius is a mean Proportional between  
" the Sine of an Arch, and the *Secant* of that  
" Arch's Complement; *that is*,

" As  $1$ ,  $OR$  : Rad.  $OC$  : : Rad.  $CF$  :  $\sqrt{e}$ ,  $c$ ,  $CN$ .

## THEOREM III.

" The Rectangles of all *Tangents*, and their  
" Complements, being respectively equal to the  
" Square of Radius are reciprocally proportional;  
" *that is*,

" As

“ As the Tangent of an Arch or  $\angle$  : Tangent  
 “ of another Arch or  $\angle$  :: Tangent of the Com-  
 “ plement of the latter Arch : Tangent of the  
 “ Complement of the former.

And because it is all one, whether of the two  
 middle Terms is put in the second or third place;  
 we may here, by varying the second Term into  
 the place of the third, compare the Tangent of  
 one Arch with the Co-tangent of another, &c.  
*that is,*

“ We may say, As  $t$  of an Arch or  $\angle$  :  $ct$ , of  
 “ another Arch ::  $t$ , of the latter Arch :  $ct$ , of the  
 former.

#### THEOREM IV.

“ The Sines of the Arches, and the Secants of  
 “ their Complements, are reciprocally propor-  
 “ tional; *that is,*

“ As  $s$ , of an Arch :  $s$ , of another Arch ::  $co-se$ ,  
 “ of the latter Arch :  $co-se$ , of the former.

“ And by changing the second and third Terms,  
 “ a Sine may be compared with a Secant.

*In any Proportion, if the two first Terms be*

*As  $t$ , of an Arch : Radius;*

*To bring the Radius into the first place, it may be said,*  
*As Rad :  $ct$ , of that Arch;*

*because there is the same Proportion between these two latter  
 Terms as between the two former. Now in all the  
 Theorems, the two latter Terms consist either of the  
 Parts, or of the Complements of the Parts of the two  
 former. Hence it will not be difficult to vary any Pro-  
 portion propounded. For*

1. From hence it will follow, that a Proportion  
 wholly in Tangents may be changed into their Com-  
 plements, without altering the Order of the Terms;  
 and the Converse. And

If

If it were, As  $t, 10^{\circ} : t, 20^{\circ} :: t, 52^{\circ} : t, 69^{\circ} 15'$ ,  
it would also be,

$$\text{As } t, 60^{\circ} : t, 70^{\circ} :: t, 38^{\circ} : t, 20^{\circ} 45'.$$

2. That if the two latter Terms of any Proportion being *Tangents* are only changed into their Complements, it infers a Transportation of the first Term into the second place. As in the Example before.

$$\text{As } t, 20^{\circ} : t, 10^{\circ} :: t, 38^{\circ} : t, 20^{\circ} 45'.$$

3. That if the two former Terms of a Proportion being *Tangents*, are changed into their Complements, it likewise infers a Change of the third Term into the place of the fourth.

And then if the fourth Term be sought, it will hold, as the second Term to the first, so is the third Term (as at first propounded) to the fourth. In the first Example,

$$\text{As } t, 70^{\circ} : t, 80^{\circ} :: t, 52^{\circ} : t, 69^{\circ} 15'.$$

4. A Proportion wholly in *Secants*, may be changed into a Proportion wholly in *Sines* without altering the Order of the Places, only by taking their Complements; and the *Converse*. So if it were,

$$\text{As } se, 80^{\circ} : se, 70^{\circ} :: se, 60^{\circ} : se, 10^{\circ}.$$

It would also hold in *Sines*,

$$\text{As } s, 10^{\circ} : s, 20^{\circ} :: s, 30^{\circ} : s, 80^{\circ}.$$

5. If the two latter Terms, being *Secants*, should be changed into *Sines*; and the *Converse*, if they were *Sines*, to be turned into *Secants*, it would be done only by taking their Complements, but then must the second and first Terms change places one with another. So if the Proportion were,

$$\text{As } s, 12^{\circ} : s, 42^{\circ} :: se, 36^{\circ} : se, 75^{\circ} 26',$$

it would also hold,

$$\text{As } s, 42^{\circ} : s, 12^{\circ} :: s, 54^{\circ} : s, 14^{\circ} 34'.$$

6. That

6. That if the two former Terms of a Proportion in *Secants* should be changed into *Sines*, and the *Converse*; this would infer a changing of the fourth Term of that Proportion into the place of the third: But the third Term not being that which is sought, the Rule to do it would be to imagine the two first Terms to change places, and then to take their Complements; so if the Proportion were,

As *se*,  $36^{\circ}$  : *se*,  $75^{\circ} 26'$  :: *s*,  $12^{\circ}$  : *s*,  $42^{\circ}$ ,  
it would also hold,

As *s*,  $14^{\circ} 34'$  : *s*,  $54^{\circ}$  :: *s*,  $12^{\circ}$  : *s*,  $42^{\circ}$ .

7. Two Terms, whether the former or latter, in any Proportion, being as a Sine to Tangent, may be varied: For, as *t*, of an Arch : *s*, of another Arch :: *co-se*, of the latter Arch : *ct*, of the former.

And by transposing the Order of the Terms, as a Sine : a Tangent :: *ct*, of the latter Arch : *co-se* of the former.

8. Lastly, Observe that if four Terms or Numbers are proportional, their Order may be so transposed, that each of those Terms may be last in Proportion: and so of any four proportional Terms, if three be given, the other that is wanting may be found.

Thus;

As, First : Second :: Third : Fourth.

As, Second : First :: Fourth : Third.

As, Third : Fourth :: First : Second.

As, Fourth : Third :: Second : First.

Thus have I explain'd the Nature of Proportion, and given sufficient Directions for varying of the same, to make it easy to a young Beginner;

B b

and



and I could wish that those who pretend to study Mathematicks would, as soon as they are able, get a clear Knowledge of these important Matters, and of every thing contain'd herein besides, before ever they attempt the Study of those *Sciences*; since 'tis impossible to arrive at any tolerable Skill in any one of them, without a previous understanding of all these Matters, which are the very *Basis* and *Foundation* of *Mathematicks*.



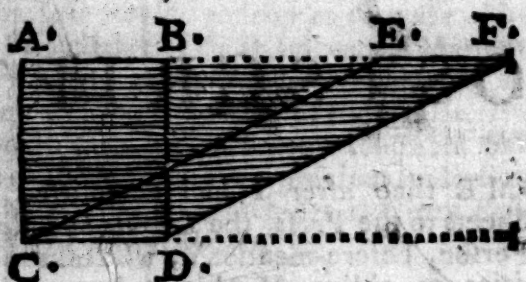
# POSTSCRIPT.

SOME time after I had committed these Sheets to the Press, the *Learned and Reverend* Person, before-mention'd, who takes all Opportunities to oblige Mankind, but especially those who have the Happiness to be known to him, was pleas'd to give me the hint, in a Letter, That the Demonstration of the 12th *Theorem* was only true in that particular Instance, according to the Method there used; and that in other Cases, the Demonstration thereof would fail; which I confess to be true, but did not observe it before. Now to make the Learner some amends for that Oversight, I shall here (by way of Postscript) present him with another Demonstration of that *Theorem*, which is very easy to apprehend, and likewise *universal*. And altho' it be in a Method very different from any of the rest, being not long since invented by *Cavalierius*; yet I the rather chuse it in this place, because if the Learner likes it, he may apply it to some other of the *Theorems* in this Book, and elsewhere, and so make his Advantage of it.

## THEOREM XII.

*Demonstrated by Cavalerius's Method of Indivisibles.*

Let there be two Parallelograms propos'd, having the same Base and Altitude, *i. e.* lying between



ween the same Parallels; as ABCD and EFCD. Then if the Line AC be divided into any Number of Parts, and small Lines drawn from every Part thereof through BD, and also through the Parallelogram FC, and all of them parallel to the Base CD, it will from a little Consideration be evident, that the two Spaces ABCD and EFCD, are equal; since the same Number of Lines of equal Length, will fill them both; for there can be no more Lines in one than in the other: neither can they be longer, or shorter, or closer, in one than in the other; since they are all equal to AB, or CD, and also parallel thereunto: So that the two Parallelograms are equal, according to this Demonstration.

N. B. This Method is very much approved of by some, and rejected by others; tho' I think it is not without its Use and Benefit to young Learners especially.



F I N I S.



